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Your Roll No. ....

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B.Sc. (Prog.)/II **C**

MA 201—MATHEMATICS—I

(Calculus and Geometry)

(For Physical Sciences/Applied Sciences)

(Admissions of 2005 and onwards)

Time : 3 *Hours* *Maximum Marks* : 112

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Section I

Attempt any *two* parts from Q. No. 1.

1. (*a*) (*i*) Give the geometrical representation of addition of two complex numbers. 5½

(*ii*) Find the equation of the circle whose radius is 3 and whose centre has affix $(1 \quad i)$. 7

P.T.O.

(*b*) (*i*) Solve the equation :

$$3x^3 + 11x^2 + 12x + 4 = 0$$

being given that the roots are in H.P. 7

(*ii*) If, α, β, γ are the roots of the equation : 5½

$$x^3 + px^2 + qx + r = 0 \text{ and } r \neq 0,$$

find the value of

$$\sum \frac{\left(\beta^2 + \gamma^2\right)}{\beta\gamma}.$$

(*c*) (*i*) Use De Moivre's theorem to solve the equation : 7

$$z^7 - z^4 + z^3 - 1 = 0.$$

(*ii*) If α, β be the roots of 5½

$$x^2 - 2x + 4 = 0,$$

prove that

$$\alpha^n + \beta^n = 2^{n+1} \cos n\frac{\pi}{3}.$$

Section II

Attempt any *two* parts from Q. Nos. **2**, **3** and **4**.

2. (*a*) (*i*) Using $\in$-δ approach show that : 5

$$\lim_{x \to 2} (3x - 5) = 1$$

(*ii*) Show that the function

$$f(x) = \begin{cases} x \dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

is not derivable at $x = 0$. 5

(*b*) (*i*) State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function : 5

$$f(x) = (x - 1)(x - 2)(x - 3) \text{ in } [1, 4]$$

(*ii*) Prove that uniform continuity implies continuity. Show by an example that the converse is not true. 5

(c) (i) Determine the intervals of concavity and points of inflexion of the curve : 5

$$a^2 y^2 = x^2(a^2 - x^2).$$

(ii) State Darboux's theorem. Explain it with the help of an example. 5

3\. (a) Find the asymptotes of the following curve : 10

$$(y - a)^2 (x^2 - a^2) = x^4 + a^4.$$

(b) (i) Find the position and nature of the double points of the curve : 5

$$y^2 = (x - a)^2 (x - b).$$

(ii) Trace the curve : 5

$$r^2 = a^2 \cos 2\theta.$$

(c) Trace the curve : 10

$$y^2(2a - x) = x^3.$$

4\. (a) If

$$I_{m,n} = \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x \, dx,$$

prove that

$$I_{m,n} = \frac{m - 1}{m + n} \cdot I_{m-2,n}$$

m and n being +ve integers. 10

(*b*) Find the area bounded by the curve : 10

$$r = a(1 + \cos\theta).$$

(*c*) Find the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x-axis. 10

Section III

Attempt *four* parts of this question. Part (*a*) is compulsory.

5. (*a*) Rotate the co-ordinate axes to get rid of the xy-term from the equation : 12

$$52x^2 - 72xy + 73y^2 + 40x + 30y - 75 = 0$$

and trace the conic.

Or

Rotate the co-ordinate axes to get rid of the xy-term from the equation :

$$x^2 + 4xy - 2y^2 - 6 = 0$$

and trace the conic. 12

(*b*) Find the equation of the parabda that has vertex $(5, -3)$, axis parallel to y-axis and passes through $(9, 5)$. 5

(c) If $\vec{A} = 2t\hat{i} + 3t^2\hat{j} + t^3\hat{k}$ and $\vec{B} = t^4\hat{k}$ 5

find .

(i) $\frac{d}{dt}\left(\vec{A} \cdot \vec{B}\right)$

(ii) $\frac{d}{dt}\left(\vec{A} \times \vec{B}\right)$.

(d) Show that : 5

$$\nabla \cdot \left(\frac{\vec{r}}{r^3}\right) = 0$$

where

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, \quad r = |\vec{r}|$$

(e) If 5

$$\vec{A} = \sin x\hat{i} + \cos(x-y)\hat{j} + z\hat{k},$$

find

$$\nabla \cdot \left(\nabla \times \vec{A}\right).$$