

This question paper contains 4 printed pages.

8059

Your Roll No.

B.Sc. (G) / II

D

MATHEMATICS — PAPER III

(Geometry)

Time : 3 hours

Maximum Marks : 55

*(Write your Roll No. on the top immediately
on receipt of this question paper)*

*Attempt all questions, selecting two parts
from each question.*

1. (a) Find the equation of the circle which passes through (2, 2) and belongs to the coaxal system of which the limiting points are (1, 2) and (3, 5).
- (b) Find the condition that the chord of contact of tangents, drawn from the point (x', y') subtends a right angle at the centre.
- (c) Define radical axis of two circles. Find the radical axis of two circles and prove that the radical axis is perpendicular to their line of centers.

9

P.T.O.

2. (a) Prove that the tangents at the extremities of a focal chord of a parabola intersect at right angles on the directrix.
- (b) Prove that the portion of a tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus.
- (c) Find the locus of poles of tangents to the circle $x^2 + y^2 = 4a^2$ with respect to the parabola $y^2 = 4ax$. 9
3. (a) Find the locus of the pole w.r.t. the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ of any tangent to the circle $x^2 + y^2 = a^2$.
- (b) If the normal at one end of a latus rectum of an ellipse passes through one extremity of the minor axis, prove that the eccentricity e of the curve is given by the equation $e^4 + e^2 - 1 = 0$.
- (c) Show that the line $x \cos \alpha + y \sin \alpha = p$ will be tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ if $a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$. 9
4. (a) Obtain the equation of the sphere circumscribing the tetrahedron whose faces are:

$$x=0, y=0, z=0; \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

- (b) Find the equation of the sphere containing the circle

$$x^2 + y^2 + z^2 + 2x - 4y + 6z + 5 = 0; \quad x + 2y + 3z - 8 = 0$$

as a great circle.

- (c) Find the limiting points of the coaxial system of spheres determined by the system:

$$x^2 + y^2 + z^2 + 3y - 3z + 6 = 0;$$

$$x^2 + y^2 + z^2 + 2x + 4y - 2z + 6 = 0$$

9

5. (a) Find the equation of the elliptic cone where vertex is at the origin and intersects the ellipse $\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and $x = a$.

- (b) Find the equations of lines of intersection of the cone $x^2 - 5y^2 + z^2 = 0$ with the plane $x - 3y + z = 0$. Also find the angle between the lines.

- (c) Find the equations of the right circular cylinder whose guiding circle is:

$$x^2 + y^2 + z^2 = r^2; \quad x + y + z = 0.$$

9

6. Trace any *one* of the following conics giving essential details:—

(a) $17x^2 - 12xy^2 + 8y^2 + 46x - 28y + 17 = 0$

(b) $x^2 + xy^2 + y^2 - x + 4y + 3 = 0$

10