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Your Roll No. ....

8060

B.Sc. (G)/II

D

MATHEMATICS—Paper IV

(Vector Calculus and Differential Equations)

Time : 3 Hours

Maximum Marks : 55

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any two parts from each question.

1. (a) Prove that :

$$\text{Div}(\text{curl } \vec{A}) = 0$$

for any vector function  $\vec{A}$ .

(b) If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $r = |\vec{r}|$ , prove that :

$$\nabla \cdot \left( \frac{\vec{r}^{-1}}{r^3} \right) = 0.$$

P.T.O.

- (c) Find the directional derivative of  $\phi = 2x^3 - 3yz$ ,  
at the point  $(2, 1, 3)$  in the direction parallel to  
 $(2, 1, -2)$ . (5,5,5)

2. (a) Solve :

$$xy^2 dx + (x^2 y^2 + x^2 y) dy = 0.$$

- (b) Solve :

$$x \log x \frac{dy}{dx} + y = 2 \log x.$$

- (c) Solve :

$$(px - y)(py + x) = h^2 p. \quad (4.5, 4.5, 4.5)$$

3. (a) Solve :

$$(D^3 + 3D^2 + 2D)y = x^2.$$

- (b) Solve :

$$(D^2 + 3D + 2)y = e^{2x} \sin x.$$

(c) If  $y_1(x)$ ,  $y_2(x)$  are any two solutions of :

$$a_0(x)y'' + a_1(x)y' + a_2(x)y = 0,$$

then show that their linear combination :

$$c_1y_1(x) + c_2y_2(x)$$

is also a solution of the given equation. (4.5,4.5,4.5)

4. (a) Solve :

$$x^2y'' - 2x(1+x)y' + 2(1+x)y = x^3.$$

(b) Solve :

$$xy'' + (1-x)y' - y = e^x.$$

(c) Solve :

$$y'' + (\cot x)y' + 4y \operatorname{cosec}^2 x = 0. \quad (4.5,4.5,4.5)$$

5. (a) Solve :

$$y'' + 2y' + 2y = 1 + x^2$$

by the method of undetermined coefficients.

(b) Solve :

$$(D^2 - 2D)y = e^x \sin x.$$

by the method of variation of parameters.

(c) Solve :

$$(x^2 D^2 - xD + 4)y = \cos(\log x). \quad (4.5,4.5,4.5)$$

6. (a) Solve the following simultaneous differential equations :

$$\frac{d^2 x}{dt^2} = 3x + 4y$$

$$\frac{d^2 y}{dt^2} = -x - y.$$

(b) Solve :

$$\frac{dx}{y^3 x - 2x^4} = \frac{dy}{2y^4 - x^3 y} = \frac{dz}{9z(x^3 - y^3)}.$$

(c) Solve :

$$(yz + 2x)dx + (zx + 2y)dy + (xy + 2z)dz = 0.$$

(4.5,4.5,4.5)