

This question paper contains 3 printed pages.

1879

Your Roll No.

B.Sc. (G) / II

E

MATHEMATICS— Paper III

(Geometry)

Time : 3 hours

Maximum Marks : 55

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*Attempt any two parts from Question Nos. 1, 2, 3, 5, 6.
Each question carries $4\frac{1}{2}+4\frac{1}{2}=9$ marks.*

*Attempt any one part from Question No. 4.
It carries 10 marks.*

1. (a) Show that the circles:

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \text{ and}$$

$$x^2+y^2+2g_2x+2f_2y+c_2=0$$

cut each other orthogonally if $2g_1g_2+2f_1f_2=c_1+c_2$.

- (b) Find the common chord of the circles $2x^2+2y^2+x+y-1=0$ and $x^2+y^2+2x+2y=0$. Is it same as their radical axis?
- (c) Determine the coaxial system of circles whose one member is $x^2+y^2-4x+6y-3=0$ and the radical axis is the line $x-y+1=0$. Find the limiting points of the system.

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2. (a) Find the axis, focus, vertex, directrix and latus rectum of the parabola

$$x^2 = 8y - 16.$$

- (b) If $t(at^2, 2at)$ and $t'(at'^2, 2at')$ are extremities of a focal chord of the parabola $y^2 = 4ax$; show that $tt' = -1$.
- (c) Find the locus of the points from which two equal tangents can be drawn to the parabola $y^2 = 4ax$.
3. (a) Find the equations of the two tangents to the ellipse $x^2 + 2y^2 = 4$ making an intercept of 2 on y -axis.
- (b) Find the equation of tangent and normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point $(a \cos \theta, b \sin \theta)$.
- (c) Find the eccentricity, the coordinates of the foci, the equation of directrices of the hyperbola $9x^2 - 16y^2 = 144$.

4. Trace the conic:

(a) $3x^2 + 8xy - 3y^2 - 40x - 20y + 50 = 0.$

Or

(b) $x^2 + 2xy + y^2 - 2x - 1 = 0.$

5. (a) Find the equations of the tangent line to the sphere $x^2+y^2+z^2+4x-2y+6z-2=0$ at the point $(1, 1, 0)$ and lying in the plane $x+y-z+5=0$.

- (b) Find the center and radius of the circle:—

$$x^2+y^2+z^2-4x-6y+2z-2=0; \quad x+y-2z=6.$$

- (c) Define radical plane of two spheres. Show that it is a plane perpendicular to the line joining their centres.

6. (a) Find the equation of the cone whose vertex is at (α, β, γ) and passes through:—

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad z=0.$$

- (b) Find the points of intersection of the line $\frac{x-12}{5} =$

$$\frac{y-7}{2} = \frac{z+1}{-1}$$
 with the cone $5x^2 - y^2 - 11z^2 = 0$.

- (c) Find the equation of the right circular cylinder whose guiding circle is:—

$$x^2+y^2+z^2-9=0, \quad x-y+z=3.$$