1880 *Your Roll No.* .....

B.Sc. (G) / II **E**

MATHEMATICS— Paper IV

(Vector Calculus and Differential Equations)

Time : 3 hours *Maximum Marks* : 55

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory. Attempt any two parts from each question. First question is of 10 marks andeach of the remaining questions carries 9 marks.

1. (a) Find the value of the constant a so that the vector:

$$V=(axy-z^3)\mathbf{i}+(a-2)x^2\mathbf{j}+(1-a)xz^2\mathbf{k}$$

have its curl identically zero.

(b) Find $(A\times\nabla)\times B$ at the point $(1, -1, 2)$, if:

$$A=xz^2\mathbf{i}+2y\mathbf{j}-3xz\mathbf{k}$$

$$B=3xz\mathbf{i}+2yz\mathbf{j}-z^2\mathbf{k}.$$

(c) Prove that

$$\nabla\times(\nabla\times A)=\nabla(\nabla\cdot A)-\nabla^2 A$$

2. (a) Solve the differential equation:

P.T.O.

$$(2x+y-1)\,dy-(x-2y+5)\,dx=0.$$

(b) Solve the following differential equations:

(i) $\frac{dy}{dx}=e^{x-y}(e^x-e^y)$

(ii) $x^2\frac{dy}{dx}+xy+\sqrt{1-x^2y^2}=0.$

(c) Solve the following differential equations, where $p=\frac{dy}{dx}$:

(i) $yp^2+(x-y)p-x=0$

(ii) $y=2px-xp^2.$

3. (a) Prove that the two solutions $y_1(x)$ and $y_2(x)$ of the differential equation:

$$\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=0$$

where P, Q are functions of x, are linearly dependent if and only if their Wronskian is identically zero:

(b) Solve:

$$(D^3-3D^2+4D-2)y=e^x+\cos x.$$

(c) Solve:

$$(D^2-2D+1)y=x^2e^{3x}.$$

4. (a) Solve:

$$(x^2D^2-3xD+5)y=x^2\sin(\log x).$$

(b) Solve:

$$x^2\frac{d^2y}{dx^2}-2(x^2+x)\frac{dy}{dx}+(x^2+2x+2)y=0.$$

(c) Solve:

$$\frac{d^2y}{dx^2}-\frac{1}{x}\frac{dy}{dx}+4x^2y=x^4.$$

5. (a) Solve the differential equation:

$$\frac{d^2y}{dx^2}-4\frac{1}{x}\frac{dy}{dx}+4y=e^x\sin x$$

by the method of undetermined coefficients.

(b) Using the method of variation of parameters, solve:

$$\frac{d^2y}{dx^2}-2\frac{1}{x}\frac{dy}{dx}+y=\frac{e^x}{x^2}.$$

(c) Solve:

$$(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{1}{x}\frac{dy}{dx}+4y=x^2.$$

6. (a) Solve the following simultaneous differential equations:

$$\frac{dx}{dt}-\frac{dy}{dt}+2x+y=0,\ \frac{dy}{dt}+5x+3y=0.$$

(b) Solve:

$$\frac{yz\,dx}{y-z}=\frac{zx\,dy}{z-x}=\frac{xy\,dz}{x-y}.$$

(c) Solve:

$$(x^2+y^2+z^2)\,dx-2xy\,dy-2xz\,dz=0.$$