[This question paper contains 4 printed pages.]

4695 *Your Roll No.* ....

B.Sc. (G)/III **AS**

MATHEMATICS – Paper V

(Real Analysis)

Time : 3 Hours. *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each question.*

1. (a) State order completeness property of real numbers. Show that the set of rational numbers is not order complete. (4½)

   (b) Define a Closed Set. Prove that the intersection of an arbitrary family of Closed Sets is a Closed Set. Is the union of an arbitrary family of Closed Sets always closed ? Justify. (4½)

   (c) Define limit point of a set $S \subset R$. Show that the set of integers 'Z' has no limit point. (4½)

2. (a) Define a Cauchy Sequence. Show that every Cauchy Sequence is convergent. (4½)

(b) State Cauchy Convergence Criterion for Sequences and hence prove that the Sequence <a_n> defined by

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$ does not converge. (4½)

(c) State monotone convergence theorem. Show that the Sequence <a_n> defined by the relation :

$$a_1 = \frac{3}{2};\ a_{n+1} = 2 - \frac{1}{a_n},\ n \geq 1$$

is convergent. (4½)

3. (a) State and prove Cauchy's n^{th} root test for an infinite series. (5)

(b) Test the convergence of the following series :

(i) $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$

(ii) $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \cdots$ (5)

(c) Define absolute and conditional convergence for an infinite series. Check the convergence of the following series :

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^3}$. $\alpha \in R$

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (5)

4. (a) Show that a function defined and continuous on a closed interval attains its Supremum there in. (4½)

(b) Define continuity of a function. Show that the function 'f' defined by :

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$$

is discontinuous at every point of R. (4½)

(c) Define uniform continuity of a function. Show that the function $f : R \to R$ defined by $f(x) = x^2$, $\forall\ x \in R$ is not uniformly continuous. (4½)

5. (a) State and prove Cauchy's Mean Value Theorem. (4½)

(b) Prove that if $a_0, a_1, \ldots, a_n$ be real numbers such that $\frac{a_0}{n+1} + \frac{a_1}{n} + \ldots + \frac{a_{n-1}}{2} + a_n = 0$, then there exists it least one real x between 0 and 1 such that

$$a_0x^n + a_1x^{n-1} + \ldots + a_n = 0$$ (4½)

(c) Obtai Maclaurin's Series expansion of the funcon $f(x) = \cos x$, $x \in R$. (4½)

6. (a) Pr/e that $1 - \frac{x^2}{2!} \le \cos x \le 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$, for all x/R. (4½)