

1478

B.A./B.Sc. (Hons.)/III A

MATHEMATICS—Paper XVII & XVIII (III)

(Discrete Mathematics)

Time : 2 Hours

Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt All the questions.

Section I

1. Attempt any two of the following :

(a) Give description of the following graph G(Fig.1)

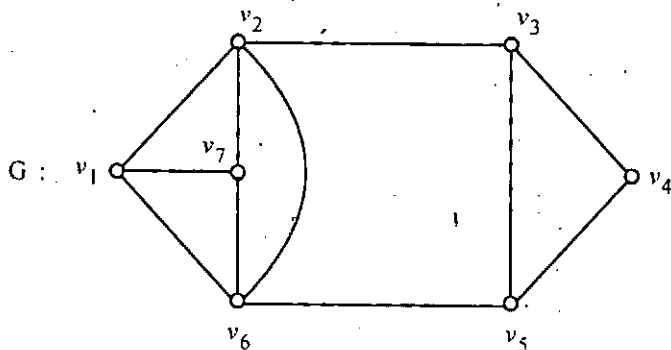


Fig. 1

Check if :

(i) G is connected.

$$(ii) G' : V' = \{v_1, v_2, v_4, v_6\}$$

$$E' = \{\{v_1, v_2\}, \{v_1, v_4\}, \{v_2, v_6\}, \{v_4, v_6\}\}$$

is a subgraph of  $G$ . Justify.

3

(b) Show that a self-complementary graph must have  $4k$  or  $4k + 1$  vertices.

3

(c) Define an Eulerian path in a graph. Find an Eulerian path in the given graph  $G$  (Fig. 2) between vertices

A and B :

3

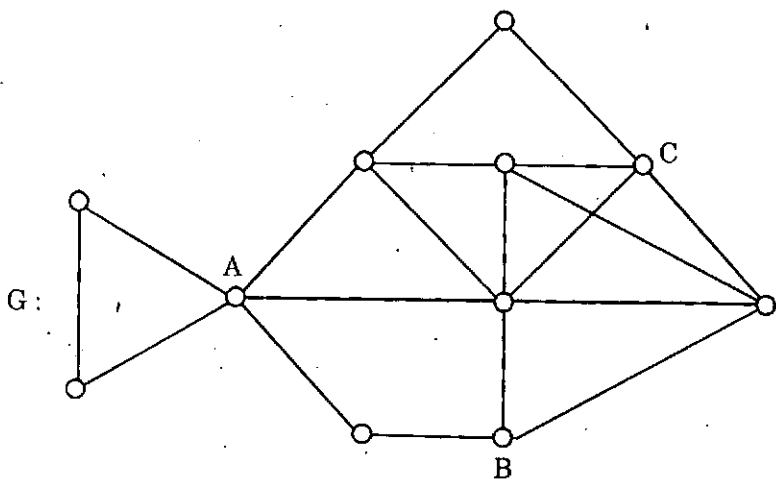


Fig. 2

2. Use nearest neighbourhood method to find a Hamiltonian circuit of minimum length for the following graph (Fig. 3), beginning at the vertex 'a'. 3½

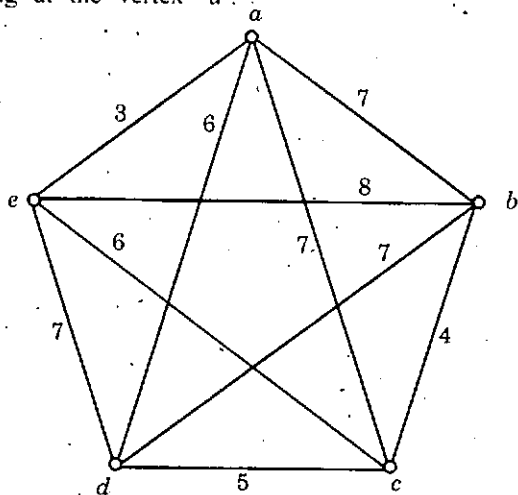


Fig. 3

## Section II

3. Attempt any two of the following : 3
- (a) Represent the following FSM in tabular form :

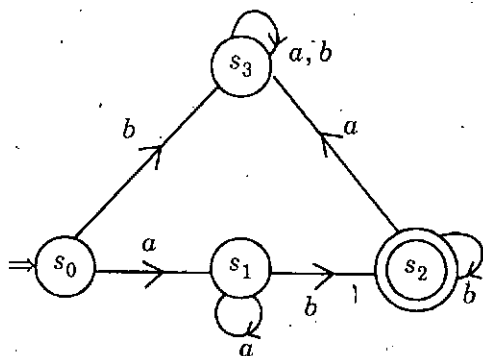


Fig. 4

Also draw the transition diagrams to find out the output sequences corresponding to the input sequences :

(i)  $a a b a a b b a$

(ii)  $b a a a b b b a$ .

(b) Design a FSM with input symbols as 0, 1 and which accepts sequences having even numbers of 1's. 3

(c) Which type/types of strings are accepted by the following FSM : 3

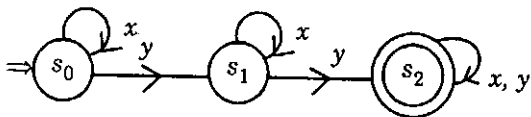


Fig. 5

4. Reduce the following FSM shown in the table, to an equivalent FSM with smallest number of states : 3½

State ↓	Input		Output ↓
	0	1	
A	B	H	0
B	F	D	0

C	A	F	0
D	A	G	0
E	D	B	1
F	C	B	1
G	D	B	1
H	C	A	0

## Section III

5. Attempt any two of the following :

(a) Show that there is no (12, 8, 3, 2, 1) configuration. 3

(b) Show that for a  $(b, v, r, k, \lambda)$  configuration

$$(i) \quad vr(k-1)\lambda = r^2(k-1)^2 + r(k-1)\lambda$$

$$(ii) \quad (k-1)\lambda = (k-1)r - (v-k)\lambda. \quad 3$$

(c) Every particle inside a nuclear reactor splits into two particles in each second. Suppose one particle is injected

into the reactor every second beginning at  $t = 0$ . How many particles are there in the reactor at the  $n$ th second ? 3

6. From the seven-point plane, construct a code system of 16 words, which can detect upto 3 errors and correct upto 1 error. 3½

#### Section IV

7. Attempt any two of the following :

- (a) Solve the recurrence relation 3

$$a_n = 4a_{n-1} - 4a_{n-2} \quad (n \geq 2)$$

$$a_0 = 1, a_1 = 3.$$

- (b) Find the generating function corresponding to the discrete numeric function : 3

$$2, 0, 2, 0, 2, 0, \dots$$

- (c) Given that  $a_0 = 0$ ,  $a_1 = 1$ ,  $a_2 = 4$ ,  $a_3 = 12$ , satisfy the recurrence relation :

$$a_r + c_1 a_{r-1} + c_2 a_{r-2} = 0,$$

determine  $a_r$ .

3

8. Solve the recurrence relation :

$$a_n = 4a_{n-1} - 4a_{n-2} + 4^n, \quad n \geq 2$$

with initial conditions  $a_0 = 2$ ,  $a_1 = 8$ . Also, obtain an expression for the corresponding generating functions.  $3\frac{1}{2}$