This question paper contains 4 printed pages.]

Your Roll No. ....

1470

B.A./B.Sc. (Hons.)/III A

MATHEMATICS – Unit XI

(Differential Equations – II)

Time : 2 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** questions in all, selecting **one** question from each Section. **Each** question carries equal marks.

SECTION – I

1. (a) Explain Charpit's method for solving the general non-linear partial differential equation of first order. 5

   (b) Find the general solution of the partial differential equation 4½

   $px(x + y) - qy(x + y) + (x - y)(2x + 2y + z) = 0$

2. (a) Show that the equations 3

   $xp = yq,\ z(xp + yq) = 2xy$

   are compatible and solve them.

(b) Find the complete integral of 3

$2(y + zq) = q(xp + yq)$

(c) Using Jacobi's method, find the complete integral of the equation 3½

$(x_2 + x_3)(p_2 + p_3)^2 + zp_1 = 0$

SECTION – II

3. (a) A function ψ satisfies the non homogeneous wave equation : 5

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} + f(x, y) = 0$$

and the initial conditions

$$\psi = \frac{\partial \psi}{\partial y} = 0. \text{ When } y = 0.$$

Apply Riemann Valterra method to show that $\psi(x, y) = \frac{1}{2} \iint_T f(u, v)\, du dv$

where T is the triangle cut out from the upper half of the uv plane by the two characteristics through the point (x, y).

(b) Reduce the equation 4½

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form and hence solve it.

4. (a) Prove that, for the equation 5

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{1}{4} z = 0$$

Green's function is

$$w(x, y, z, n) = J_0 \sqrt{(x - z)(y - \eta)}$$

where $J_0(z)$ denotes Bessel's function of first kind of order zero.

(b) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form. 4½

SECTION – III

5. (a) Solve the three dimensional wave equation

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2}$$

by the method of separation of variables. 4½

(b) Derive the solution of two dimensional harmonic equation

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0 \text{ in the region.}$$

$0 \leq r \leq a,\ 0 \leq \theta \leq 2\pi$ satisfying the conditions :

(i) V remains finite as $r \to 0$.

(ii) $V = \sum_n C_n \cos(n\theta)$ on $r = a$. 5

6. (a) A gas is contained in a rigid sphere of radius a. Show that if c is the velocity of sound in the gas, the frequency of purely radial oscillations are $c\,\zeta_i/a$, where $\zeta_1, \zeta_2, \ldots$ are the positive roots of the equation $\tan\zeta = \zeta$. 4½

(b) Find the solution of the equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0 \text{ in the region}$$

$0 \leq x \leq a,\ 0 \leq y \leq b,\ 0 \leq z \leq c$ satisfying the boundary conditions.

(i) $\psi = 0$ on $x = 0, x = a, y = 0, y = b, z = 0$

(ii) $\psi = f(x, y)$ on $z = c,\ 0 \leq x \leq a,\ 0 \leq y \leq b$ 5

SECTION – IV

7. (a) Solve by Monge's method **5**

$z(qs - pt) = pq^2$

(b) Solve $(D - D'^2)z = \cos(x - 3y)$ **4½**

8. (a) Solve by Monge's method **5**

$(x - y)(x^2r - 2xys + y^2t) = 2xy(p - q)$

(b) Solve **4½**

$(x^2D^2 - xyDD' - 2y^2D'^2 + xD - 2yD')z = \log\frac{y}{x} - \frac{1}{2}$.