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Your Roll No.....

1473

B.A./B.Sc. (Hons.)/III A

MATHEMATICS—Paper XIV

(Mechanics–II)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *All* questions,

selecting *two* parts of each question.

1. (*a*) A point P moves so that it always has a velocity component of V_1 parallel to a fixed line and V_2 in the transverse direction perpendicular to OP, where O is a fixed point. Prove that its orbit is a conic having O as focus and discuss the cases :

$V_1 > V_2$, $V_1 = V_2$, $V_1 < V_2$ 4½

P.T.O.

(*b*) Show that the mass center of a system moves like a particle, having mass equal to the total mass of the system, acted on by a force equal to the vector sum of the external forces acting on the system. 4½

(*c*) A light string is attached to a fixed point O and carries at its free end a particle of mass m. The particle is describing complete revolutions about O under gravity and the string is just taut when the particle is vertically above O. Find the tension in the string when in a horizontal position. 4½

2. (*a*) A particle, moving in a straight line, is subject to a retardation of amount kv^n per unit mass, where k is a positive constant, and v is the speed at time t. Show that, if $n < 1$, the particle will come to rest at a distance $\frac{u^{2-n}}{k(2-n)}$ from the point of projection at time $t = \frac{u^{1-n}}{k(1-n)}$, where u is the initial speed. 5

What happens when

(*i*) $1 < n < 2$

(*ii*) $n > 2$.

(*b*) Mud is thrown off from the tyre of a wheel of radius 'a' of a car travelling at a speed V, where $V^2 > ga$. Neglecting the resistance of the air, show that no mud can rise higher than a height $a + \frac{V^2}{2g} + \frac{ga^2}{2V^2}$ above the ground. 5

(*c*) A heavy particle of mass m hangs at the lower end of a light vertical elastic spring of natural length l_0 and modulus $2mg$. At zero time the system is in equilibrium and the upper end of the spring is made to execute vertical oscillations so that its downward displacement at time t later is $a \sin pt$, where $p^2 = \frac{2g}{l_0}$.

Find the displacement of the particle at this instant. 5

3. (*a*) Show that the orbit described under a central attractive force varying directly as the distance is an ellipse having its center at the center of force. 5

(*b*) If $P = \mu (u^2 - au^3)$ where $a > 0$, be the central force per unit mass and a particle is projected from an apse at a distance 'a' from the center of force with a velocity $\sqrt{\frac{\mu c}{a^2}}$, where $a > c$, prove that the other apsidal distance of the orbit is $\frac{a(a+c)}{(a-c)}$. 5

(*c*) A particle of mass m moves in a central field of attractive force of which the intensity is $mkr^{-1}e^{-r^2}$, where k is a constant. Prove that a circular orbit of radius r is stable if and only if $r^2 < \frac{1}{2}$. 5

4. (*a*) A rectangular lamina ABCD is such that AB = $2a$, BC = $2b$. Find the directions of the principal axes at A. 4½

(*b*) Show that a square plate of side $2b$ suspended from one corner oscillates in unison with simple pendulum of length $\left(\frac{4\sqrt{2}}{3}\right)b$. 4½

(*c*) Find an expression for the kinetic energy of a rigid body with a fixed point rotating with angular velocity $\vec{w}$. 4½