This question paper contains 3 printed pages.]

Your Roll No. ....................

# 1477

## B.A./B.Sc. (Hons.) / III A

## MATHEMATICS – Paper XVII and XVIII (*ii*)

## (Boolean Algebra)

**Time : 2 Hours** **Maximum Marks : 38**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

Attempt any **two** parts from each Section.

Marks for each part of a question are indicated.

Any of the symbol (+) and ($\cdot$)

or ($\vee$) and ($\wedge$) may be used.

### SECTION – I

1. (a) Define a lattice in terms of an algebra; show that it is also a partially ordered set in which each pair of elements possesses a supremum and an infimum. 5

(b) Show that there are 15 abstract lattices of six elements. 5

(c) In the lattice of natural numbers ordered by divisibility, show that the set of all powers of a fixed prime $\{p^r : r = 0, 1, 2, ---\}$ is an ideal. 5

## SECTION – II

2. (a) Show that the lattices isomorphic as algebras are isomorphic as partially ordered sets and conversely. 5

(b) Let H be the image of a lattice h under a homomorphism $\theta$. If $x > y$ in H and $a \in h$ is a pre-image of $x$ in $\theta$, then $\exists$ in h at least one element b, a pre image to y and such that $a > b$ in h. 5

(c) Show that a non modules lattice must contain a sub-lattice isomorphic with the pentagonal lattice. 5

## SECTION – III

3. (a) (i) If a, b, c are elements of a modulae lattice with unity element u and if $a \vee b = (a \wedge b) \vee c = u$, then show that $a \vee (b \wedge c) = b \vee (c \wedge a) = c \vee (a \wedge b) = u$. 2

(ii) Show that a complemented modulae lattice is relatively complemented. 3

(b) (i) Show that a lattice is distributive iff for every set of elements a, b, c in L, the inequalities 2
$a \wedge c \leq b \leq a \vee c$ imply that
$(a \wedge b) \vee (b \wedge c) = b = (a \vee b) \wedge (b \vee c)$.

(ii) Let B be a Boolean Algebra, show that the map $\psi(x) = x'$ is a dual automorphism of h. 3

(c) Let h be a Boolean Algebra, show that

$$h \cong [0, a] \times [a, u], a \in h.$$ 5

## SECTION – IV

4. (a) Find the Boolean Expression (in CN form) that defines a function given by : 4

| $x$ | y | z | f |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(b) (i) Write the function $x \vee y'$ in DN form in three variables $x$, y and z. 2

(ii) Put the function $f = [(x \wedge y')' \wedge z'] \wedge [x' \vee z']'$ in DN form. 2

(c) Draw a three terminal circuit to represent the functions :

$f = (a \wedge c \wedge d) \vee (b \wedge c \wedge d)$

$g = [(a \wedge x) \vee (b \wedge x)] \wedge (y \vee z)$ 4