This question paper contains 4 printed pages.]

Your Roll No. ....

1479

B.A./B.Sc. (Hons.)/III A

MATHEMATICS – Paper XVII and XVIII *(iv)*

(Integral Transforms and Boundary Value Problems)

Time : 2 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any **two** parts from each question.

The symbols used have their usual meaning.

Section – I

1. (a) Evaluate :

(i) $L\left(\frac{1-e^{-t}}{t}\right)$.

(ii) $L^{-1}\left(\frac{1}{s^2-5s+6}\right)$.

(b) State the convolution theorem for inverse Laplace transforms. Hence, evaluate

$$L^{-1}\left(\frac{f(s)}{s}\right),$$

where $L^{-1}(f(s)) = F(t)$.

(c) By using Laplace transforms, solve the initial value problem :

$y''(x) + 2y'(x) + 5y(x) = e^{-x} \sin x$, $y(0) = 0$, $y'(0) = 1$. **4½, 4½, 4½**

Section – II

2. (a) Find the Fourier series corresponding to the function :

$$f(x) = \begin{cases} x, & 0 < x < \pi/2 \\ \pi - x, & \pi/2 < x < \pi \end{cases}$$

(b) Solve the following boundary value problem for the transverse displacements in a string :

$y_{tt}(x, t) = a^2 y_{xx}(x, t)$, $0 < x < 1$, $t > 0$,

$y(0, t) = 0$; $y(1, t) = 0$,

$y(x, 0) = \sin \pi x$; $y_t(x, 0) = 0$

(c) A string, stretched between the fixed points 0 and π on the x-axis, is initially straight with prescribed distribution of velocities $y_t(x, 0) = \sin x$. Write the boundary value problem in $y(x, t)$ and solve it. **5, 5, 5**

Section – III

3. (a) If $u(x)$ denote the steady-state temperatures in a slab bounded by the planes $x = 0$ and $x = c$ when those faces are kept at fixed temperature $u = 0$ and $u = u_0$, respectively, set up the boundary value problem for $u(x)$ and solve it.

(b) Use the method of separation of variables to solve the following boundary value problem for the temperature $u(x, t)$ in an infinite slab of material bounded by the planes $x = 0$ and $x = 1$:

$$u_t(x, t) = ku_{xx}(x, t);\ 0 \leq x \leq 1,\ t > 0,$$

$$u_x(0, t) = 0;\ u_x(1, t) = 0,\ t > 0$$

$$u(x, 0) = x,\ 0 \leq x \leq 1,$$

where K is a constant.

(c) Solve the following boundary value problem for temperatures $u(x, t)$ in an infinite slab of material :

$$u_t(x, t) = ku_{xx}(x, t),\ 0 < x < \pi;\ t > 0,$$

$$u(0, t) = 0;\ u(\pi, t) = u_0,$$

$$u(x, 0) = 0,$$

where u_0 is a constant. 5, 5, 5

Section – IV

4. (a) State and prove the Fourier integral theorem.

(b) By using the Fourier integral theorem, show that if

$$f(x) = \begin{cases} \sin x \,; & 0 \le x \le \pi \\ 0 \,; & x < 0, x > \pi, \end{cases}$$

then

$$f(x) = \frac{1}{\pi} \int_0^\infty \frac{\cos \lambda x + \cos [\lambda (\pi - x)]}{1 - \lambda^2} \, d\lambda,$$

$-\infty < x < \infty$.

Hence show that

$$\int_0^\infty \frac{\cos (\lambda \pi / 2)}{1 - \lambda^2} \, d\lambda = \frac{\pi}{2}.$$

(c) Find the Fourier cosine transform of the function :

$$f(x) = \begin{cases} x \,; & 0 < x < ½, \\ 1 - x \,; & ½ < x < 1, \\ 0 \,; & x > 1. \end{cases}$$

Also write the inverse transform. **4½, 4½, 4½**
