[This question paper contains 4 printed pages.]

2525 *Your Roll No.* .....

B.Sc. (G)/III **A**

MATHEMATICAL SCIENCES (STATISTICS)

Paper VI – Sample Surveys and Design of Experiments

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** *questions in all, selecting* **two** *from Section* **A** *and* **B** *each. Section* **C** *is compulsory.*

SECTION A

1. (a) Explain the terms : sampled population, target population, sampling unit, frame.

   (b) Define SRS. Derive the variance of the sample mean based on srswor. Also give an estimate of this variance. (2,5½)

2. (a) Prove in usual notations :

   $V(\bar{y}_{st})_{opt} \leq V(\bar{y}_{st})_{prop} \leq V(\bar{y}_{n})_{SRS}$,

   stating the assumptions made, if any.

P.T.O.

(b) Derive, to the first approximation, the expected value of the ratio estimator and the mean square error. (3½,4)

3. (a) Compare regression estimator with ratio estimator and simple random sample mean (Assume the formulae for the variance of the estimators).

(b) Prove that

$$V\left(\bar{\bar{y}}_{n1}\right) \simeq \frac{S^2}{nM}\left\{1 + \overline{M-1}\ \rho\right\},$$

where the notations have their usual meaning. (2½,5)

SECTION B

4. (a) Explain briefly the three principles of Design of Experiments.

(b) In an m×m LSD with model

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ijk}$$

where symbols have their usual meaning, find least squares estimates of μ, α_i, β_j and γ_k. Obtain variances of $\hat{\mu}$, $\hat{\alpha}_i$, $\hat{\beta}_j$ and $\hat{\gamma}_k$. (3,4½)

5. (a) What is missing plot technique? Estimate a missing value in RBD.

(b) Present the analysis of covariance for CRD. (2,5½)

6. (a) Explain Yates' technique for

(i) estimating main and interaction effects

(ii) calculating sum of squares due to these effects

in the case of 2^3 factorial design with r replications. Also write down analysis of variance table.

(b) Define a BIBD. Prove that for a symmetric BIBD for an even number of treatments to exist, $(r - \lambda)$ must be a perfect square. (4½,3)

SECTION C

7. (a) Obtain the least squares estimates of β_0 and β_1 in the model

$$y = \beta_0 + \beta_1 x + t$$

stating the underlying assumptions. Prove that these estimates are unbiased and have minimum variance.

(b) Write a note on test for 'Lack of Fit' in linear regression.

OR

Construct 95% confidence intervals for β_0 and β_1 as given in the model

$$y = \beta_0 + \beta_1 x + \epsilon \qquad (6,2)$$