[This question paper contains 4 printed pages.]

2512 *Your Roll No.* ....

B.Sc. (G)/III **A**

MATHEMATICS – Paper VI (*i*)

(Mechanics)

Time : 3 Hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each question.* **All** *questions carry equal marks.*

1. (a) A ladder leans against a smooth wall. The lower end resting on a rough road for which the coefficient of friction is $\frac{1}{4}$. Find the inclination of the ladder to the vertical when it is just on the point of slipping. (5)

(b) Forces of magnitude 3p, 7p and 5p act along the sides AB, BC and CA of an equilateral ΔABC. Find the magnitude, direction and line of action of the resultant. (5)

(c) A light rigid rod of length 2b terminated by heavy particles of weight 'w' and 'W' is placed inside a smooth hemispherical bowl of radius 'a' which is fixed with its rim horizontal. If particles of weight w rests just below the plane of the bowl, prove that $wa^2 = W(2b^2 - a^2)$. (5)

2. (a) Find the mass centre of a wire bent into the form of an isosceles right angled triangle. (4½)

(b) A uniform square lamina rests in equilibrium in a vertical plane under gravity with its sides in contact with smooth pegs in the same horizontal line at a distance 'c' apart. Show that the angle θ made by the side of a square with the horizontal in a non-symmetrical position of equilibrium is given by $c(\sin\theta + \cos\theta) = a$; 2a being the side of a square. (4½)

(c) Find the C.G. of the areas between the curves $y^2 = bx$ and $x^2 = ay$. (4½)

3. (a) A particle is moving with S.H.M. of amplitude 'a' and periodic time 'T'. Prove that

$$\int_0^T v^2\,dt = \frac{2\pi^2 a^2}{T}$$ (4½)

(b) In a S.H.M., the velocities at distances a, b, c from a fixed point on the straight line of motion (not coinciding with the centre of force) are u, v, w respectively. Prove that the time period T is given by

$4\pi^2 (b-c)(c-a)(a-b) = T^2 \sum(b-c)u^2$. (4½)

(c) A train starting at time $t = 0$, has moved in time t, a distance $x = At(1 - e^{-Bt})$, where A and B are positive constants. Find its velocity and acceleration; what do these become after a long time has elapsed ? (4½)

4. (a) If 'α' be the angle between the tangents at the extremities of any arc of a parabolic path, v, v_1, the velocities, at these extremities, and u the horizontal component of the velocity, show that the time of describing the arc is

$$\frac{vv_1 \sin\alpha}{gu}$$
(4½)

(b) If particles are projected from the point 0 in a vertical plane under gravity with velocity $\sqrt{2gK}$, prove that the locus of the vertices of their paths is the ellipse $x^2 + 4y(y - K) = 0$. (4½)

(c) A gun is fired from a moving platform, and the ranges of the shot are observed to be R and S, when the platform is moving forward and backward, respectively, with velocity V. Prove that the elevation of the gun is $\tan^{-1}\left[\frac{g(R-S)^2}{4V^2(R+S)}\right]$. (4½)

5. (a) A bead of mass 'm' is projected with velocity 'u' from the lowest point A of a smooth fixed vertical circle of radius 'r'. What is the velocity when it is at B, where $\angle AOB = \theta$? (4½)

(b) A light in extensible string is attached to a fixed point 0, and carries at its free end, a particle of mass m. The particle is describing complete revolutions about 0 under gravity, and the string is just taut when the particle is vertically above 0. Find the tension in the string, when in a horizontal position. (4½)

(c) A motor car weighing 10 quintals and travelling at 12 m/sec is brought to rest in 18 metres, by the application of its brakes. Find the work done by the force of resistance due to brakes. (4½)

6. (a) A rectangular area is immersed in a heavy liquid with two sides horizontal, and is divided by horizontal lines into strips on which the total thrust are equal. Prove that if a, b, c are the breaths of three consecutive strips, then

$$a(a + b)(b - c) = c(b + c)(a - b). \quad (4½)$$

(b) Find the centre of pressure of a regular hexagon of side 'a', with one side in the surface. (4½)

(c) A parallelogram has the highest angular point in the surface of the liquid and one diagonal horizontal. Show that its depth of centre of pressure is $\frac{7}{12}$ of the depth of the lowest point. (4½)