[This question paper contains 4 printed pages.]

**2524** *Your Roll No.* ..............

**B.Sc. (Gen.)/III** **A**

MATHEMATICAL SCIENCES (STATISTICS) – Paper V

Statistical Inference

*Time : 3 Hours* *Maximum Marks : 38*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Attempt any* **Four** *questions.*
**All** *questions carry equal marks.*

1. (a) Examine the unbiasedness of the following estimators :

(i) $S_1^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

(ii) $S_2^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \mu\right)^2$ (where $\mu$ is known)

for $\sigma^2$, the population variance.

(b) If $X_1, \ldots, X_n$ are random observation on a Bernoulli variate X taking the value 1 with probability p and the value 0 with probability $(1-p)$, show that

(i) $\frac{\sum_{i=1}^{n} x_i}{n}\left(1-\frac{\sum_{i=1}^{n} x_i}{n}\right)$ is a consistent estimator of $p(1-p)$.

(ii) $\frac{\left[\sum_{i=1}^{n} x_i\left(\sum_{i=1}^{n} x_i - 1\right)\right]}{n(n-1)}$ is an unbiased estimate of $p^2$.

2. (a) State and prove the Cramer-Rao Inequality.

(b) A random sample $x_1, ..., x_n$ is taken from a normal population with mean zero and variance $\sigma^2$. Examine if $\sum_{i=1}^{n} x_i{}^2/n$ is an MVB estimator for $\sigma^2$.

3. (a) In random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimators for

(i) $\mu$ when $\sigma^2$ is known

(ii) $\sigma^2$ when $\mu$ is known

(iii) $\mu$ and $\sigma^2$

(b) If $x_1, ..., x_n$ is a random sample from a distribution

$$f(x, \theta) = \theta x^{\theta-1} \quad 0 < x < 1,$$

show that $Y = \prod_{i=1}^{n} x_i$ is a sufficient statistic for $\theta$.

(c) On the basis of a random sample of size n drawn from $N(\theta, \sigma^2)$, obtain $100(1-\alpha)\%$ confidence interval for :

(i) $\theta$ when $\sigma^2$ is known

(ii) $\theta$ when $\sigma^2$ is unknown

(iii) $\sigma^2$ when $\theta$ is known

4. (a) Explain how the sequential test procedure differs from the Neyman-Pearson test procedure. Describe S.P.R.T. and its OC and ASN functions.

(b) Explain how the run test can be used to test the randomness of a given set of observations.

(c) Let p be the probability that a coin will fall head in a single toss. In order to test $H_0 : p = 1/2$ against $H_1 : p = 3/4$ the coin is tossed 5 times and $H_0$ is rejected if more than 3 heads are obtained. Find the probability of type I error and power of the test.

5. (a) Use Neyman-Pearson Lemma to obtain the best critical region for testing $\theta = \theta_0$ against $\theta = \theta_1$ in the case of a normal population $N(\theta, \sigma^2)$ where $\sigma^2$ is known. Hence find the power of the test.

(b) Explain the main differences between the parametric and non parametric approaches to the theory of statistical inference. What are the advantages of the non parametric tests ?

6. Write short notes on any **three** :

(i) Method of moments

(ii) Median Test

(iii) Rao-Blackwell Theorem

(iv) Concepts of statistical hypothesis, critical region, best critical region, parametric space and sample space