16/8/11 3(a) Corrections

[This question paper contains 4 printed pages.]

624 *Your Roll No.* ....

B.Sc. Prog./III **A**

MP–301 : Mathematics – II

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** *is compulsory.*
Attempt any **two** *parts from Question* **2** *of* **Unit I.**
Attempt any **two** *questions from* **Unit II** *and any* **two** *questions from* **Unit III.**

1. (a) If Σu_n and Σv_n converge to u and v respectively, then show that $\Sigma(u_n + v_n)$ converges to $(u + v)$. Does the converse holds true ? Justify. (4)

   (b) Solve the differential equation :

   $(D^2 + 3D + 2)y = e^{2x} \sin x$ (4)

   (c) If H and K are subgroups of an abelian group G, then HK is also a subgroup of G. (4)

P.T.O.

UNIT I

2. (a) Let $\langle a_n \rangle$ be a sequence defined as

$$a_1 = 1,\ a_{n+1} = \left(\frac{3 + a_n^2}{2}\right)^{1/2},\ n \geq 1.$$

Show that $\langle a_n \rangle$ converges and also, find $\lim\limits_{n \to \infty} a_n$. (12)

(b) State D' Alembert's Ratio Test for convergence of series of positive terms. Also test the following series for convergence :

$$\frac{1}{3} + \frac{2!}{9} + \frac{3!}{27} + \frac{4!}{81} + \ldots\ldots$$ (12)

(c) Define an alternating series. State and prove Leibnitz's test for the convergence of an alternating series. (12)

UNIT II

3. (a) Solve the following differential equation by method of variation of parameters

$$(D^2 + \mu)y = \sec 2x.$$ (10)

(b) Solve $(D^2 - 5D + 6)y = e^{4x}(x^n + 9)$. (10)

4. (a) Solve : $\dfrac{dx}{\cos(x+y)} = \dfrac{dy}{\sin(x+y)} = \dfrac{dz}{z}$. (10)

(b) State Lagrange's method for solving quasilinear partial differential equations of first order. Also, solve

$$y^2zp + x^2zq = y^2x \quad (10)$$

5. (a) A 6 lb weight is attached to the lower end of a coil spring suspended from the ceiling, the spring constant of the spring being 27 lb/feet. The weight comes to rest in its equilibrium position and beginning at $t = 0$ an external force given by $12 \cos 20t$ is applied to the system. Determine the resulting displacement assuming damping is negligible. (10)

(b) (i) Find the complete integral of the equation
$Z = px + qy + p^2 + q^2$. (5)

(ii) Classify the second order partial differential equation

$$r - x^2t = 0 \quad (2)$$

(iii) By eliminating the arbitrary constants a and b, form a partial differential equation from the equation

$$Z = (a + x)(b + y) \quad (3)$$

UNIT III

6. (a) State Cayley Hamilton Theorem and verify it for the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & -3 \\ 2 & 1 & -1 \end{pmatrix} \quad (9)$$

(b) Find eigen values and eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & 0 & 3 \end{pmatrix}$$ (9)

7. (a) Show that if for every two elements a and b of a group G, $(a.b)^2 = a^2.b^2$, then G is an abelian group. (9)

(b) Prove that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$; for all $a, b \in H$. (9)

8. (a) State and prove Lagrange's theorem for finite groups. (9)

(b) Let $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} : a \in \mathbb{R}, a \neq 0 \right\}$.

Show that G is a group under matrix multiplication. (9)