[This question paper contains 4 printed pages.]

4709 *Your Roll No.* ....

B.Sc. (G)/III **AS**

MATHEMATICAL SCIENCES (STATISTICS)

Paper VI – Sample Surveys and Design of Experiments

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **five** *questions in all, selecting* **two** *from Section* **A** *and* **B** *each. Section* **C** *is compulsory.*

SECTION A

1. (a) What are non-sampling errors ? How do they differ from sampling errors ?

   (b) To estimate the population mean in case of simple random sampling without replacement, describe the method of determining the sample size with given (i) coefficient of variation β, (ii) margin of error d with confidence coefficient (1–α) and (iii) margin of error d as a fraction of true value. (2½,4½)

2. (a) In usual notations, prove that

$$v(\bar{y}_{st})_{opt} \le v(\bar{y}_{st})_{prop} \le v(\bar{y}_n)_{SRS}.$$

P.T.O.

(b) With two strata, a sampler would like to have $n_1 = n_2$ for administrative convenience, instead of using the values given by Neyman's allocation. If $v(\bar{y}_{st})$ and $v(\bar{y}_{st})_{opt}$ denote the variances given by $n_1 = n_2$ and the Neyman's allocation, show that the fractional increase in variance is

$$\frac{v(\bar{y}_{st}) - v(\bar{y}_{st})_{opt}}{v(\bar{y}_{st})_{opt}} = \left(\frac{r-1}{r+1}\right)^2,$$

where $r = n_1/n_2$ as given by Neyman's allocation.

(4½,2½)

3. (a) Calculate bias of a regression estimator to the first approximation.

(b) Compare regression estimator with ratio estimator and with simple random sample mean, assuming the formulae for the variance of the estimators.

(3½,3½)

4. (a) Prove that

$$v(\bar{y}_{nm}) = \left(\frac{1}{n} - \frac{1}{N}\right)S_b^2 + \left(\frac{1}{m} - \frac{1}{M}\right)\bar{S}_w^2,$$

the notations have their usual meanings.

(b) Prove that the efficiency of cluster sampling *relative to* simple random sampling increases as mean square within clusters increases. (3½,3½)

SECTION B

5. (a) Prove that the analysis of covariance, with one concomitant variable, when applied to RBD reduces the error sum of squares.

   (b) Define the terms : randomisation, local control, replication and uniformity trial. (4,4)

6. (a) Explain the technique of analysis of variance.

   (b) Derive analysis of variance of an LSD with one missing observation. (2½,5½)

7. (a) Present the analysis of 2^n factorial design.

   (b) Explain the concept of complete and partial confounding. Construct a 2^4 factorial experiment, the confounding effect being ABCD. (4,4)

8. (a) Define a BIBD. When do you call a BIBD (i) symmetric and (ii) resolvable ?

   (b) Prove that in a resolvable BIBD,

$$b \geq v + r - 1.$$ (4,4)

SECTION C

9. For the simple linear regression model

$$y = \beta_0 + \beta_1 x + \epsilon$$

with $\in \sim NID(0, \sigma^2)$, σ^2 being unknown, obtain the least squares estimators for β_0 and β_1 along with their variances. Show that

$$Cov\left(\hat{\beta}_0, \hat{\beta}_1\right) = -\frac{\bar{x}\sigma^2}{S_{xx}}.$$

Also verify the bias and variance properties of $\hat{\beta}_0$ and $\hat{\beta}_1$. (8)