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4706

Your Roll No.

B.Sc. (G)/III

AS

MATHEMATICAL SCIENCES (STATISTICS)

Paper III – Statistical Methods – II

Time : 3 Hours

Maximum Marks : 38

(Write your Roll No. on the top immediately
on receipt of this question paper.)

Attempt any six questions.

All questions carry equal marks.

1. (a) If X_1 and X_2 are two independent chi-square variates with n_1 and n_2 d.f., respectively, then prove that

$$\frac{X_1}{X_1 + X_2} \sim \beta_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right).$$

- (b) Show that for large d.f., chi-square distribution tends to normal distribution.

2. (a) If X follows t-distribution with n d.f., prove that

$$\frac{\mu_{2r}}{\mu_{2r-2}} = \frac{n(2r-1)}{(n-2r)}, \quad n > 2r.$$

Hence find β_1 and β_2 .

- (b) Define F-statistic and show that mode of F-distribution is always less than unity.

P.T.O.

3. (a) Show that for t-distribution with n d.f. mean deviation about mean is given by

$$\sqrt{n} \sqrt{(n-1)/2} / \sqrt{\pi} \sqrt{n/2}$$

- (b) In one sample of 8 observations, the sum of the squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level, given that $F_{7,9}(0.05) = 3.29$.

4. (a) In a large consignment of oranges a random sample of 64 oranges revealed that 14 oranges were bad. Is it reasonable to assume that 20% of the oranges were bad?

- (b) Obtain the expression for an unbiased estimate of the population variance σ^2 .

5. (a) Explain the terms :

- (i) Type I and Type II errors
- (ii) Critical region

- (b) In a certain experiment to compare two types of animal foods A and B, the following results of increase in weights were observed in animals :

Animal no.	1	2	3	4	5	6	7	8
Increase Food A in wt.	49	53	51	52	47	50	52	53
Food B	52	55	52	53	50	54	54	53

Can we conclude that food B is better than food A when the same set of eight animals were used in both the foods.

(Given that $p(t > 1.90) = .05$ for 7 d.f.)

6. (a) A dice is thrown 60 times with the following results :

Face	:	1	2	3	4	5	6
Frequency	:	8	7	12	8	14	11

Test at 5% level of significance if the dice is unbiased, assuming that $P(\chi^2 > 11.1) = .05$ with 5 d.f.

- (b) Obtain the distribution function and hence the p.d.f. of the smallest sample observation $X_{(1)}$ in a random sample of size n from a population with a continuous distribution function $F(x)$. Show that for random sample of size 2 from normal population $N(0, \sigma^2)$, $E(\chi_{(1)}^2) = -\sigma/\sqrt{\pi}$.

7. (a) A sample of 400 observations has mean 95 and standard deviation 12. Can it be regarded as a random sample from a population with mean 98? What can be the maximum value of the population mean?

P.T.O.

- (b) Explain Logarithmic and Sin^{-1} transformations.
8. (a) What is the relation between χ^2 and F distributions ?
- (b) Derive the expressions for the standard error of
- (i) the mean of a random sample of size n , and
 - (ii) the difference of the means of two independent random samples of sizes n_1 and n_2 .