[This question paper contains 6 printed pages.]

Your Roll No. .....

5012 **B**

B.Sc. (G)/III

MATHEMATICS – Paper VI (*i*)

(Mechanics)

Time : 3 *Hours* *Maximum Marks* : 55

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) ABCD is a square of side '*a*' metre in length. Forces of magnitudes 2, 3 and 6 kg, act along AB, BC and CD respectively. Reduce the system to a single force acting at C, together with a couple. 4½

(*b*) A uniform ladder rests with its lower end on a rough horizontal ground and its upper end against a smooth vertical wall. Prove that a horizontal force applied at the foot of the ladder to make it move towards the wall must be at least

$$W\left(\mu + \frac{1}{2}\tan\theta\right),$$

where W is the weight of the ladder, θ is its inclination to the vertical and μ is the coefficient

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of friction between the foot of the ladder and the ground. 4½

(*c*) A string ABCD hangs from fixed points A, D carrying a weight of 12 lb at B and a weight W at C. AB is inclined at 60° to the horizontal, BC is horizontal and CD is inclined at 30° to the horizontal. Find W. 4½

2. (*a*) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined at their extremities so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Prove that the tension in the string is 3W. 4½

(*b*) Find the centre of gravity of the area bounded by the parabola $y^2 = 4ax$, the x-axis and the latus rectum of the parabola. 4½

(*c*) Forces P, 2P, 3P, 4P, 5P and 6P respectively act at the vertices of a regular hexagon inscribed in a circle of radius R, in a direction perpendicular to the plane of the hexagon and in the same sense.

Show that their resultant cuts the plane of the hexagon at a point distant $\frac{2R}{7}$ from the centre. 4½

3. (*a*) A simple pendulum of mass *m* and length *a* is hanging in equilibrium. At $t = 0$, a small horizontal disturbing force X comes into operation and continues to act, varying with time according to the formula

$$X = mb \sin 2pt,$$

where $ap^2 = g$.

Find a formula giving the position of the pendulum at any time. 4½

(*b*) A particle of mass *m* is placed on a horizontal board which is made to execute vertical simple harmonic oscillations of period T and amplitude *a*. If

$$a < \frac{gT^2}{4\pi^2},$$

show that the particle does not lose contact with the board at any time. 4½

(*c*) A particle is executing S.H.M. of amplitude *a* and time period T. Prove that : 4½

$$\int_0^T v^2 \, dt = \frac{2\pi^2 a^2}{T}.$$

4. (*a*) A gun is mounted on a hill of height h above a level plane. Show that if the resistance of the air is neglected, the greatest horizontal range for given muzzle velocity V is obtained by firing at an angle of elevation θ, such that : 4½

$$\tan^2 \theta = \frac{V^2}{V^2 + 2gh}.$$

(*b*) A particle just clears a wall of height 'b' at a distance 'a' and strikes the ground at a distance 'c' from the point of projection. Prove that the angle of projection is : 4½

$$\tan^{-1}\left[\frac{bc}{a(c-a)}\right].$$

(*c*) Find the greatest distance that a stone be thrown inside a horizontal tunnel 10 ft high, with a velocity of projection 80 ft/sec. Also, find the corresponding time of flight. 4½

5. (*a*) A *particle* is placed on the outside of a smooth vertical circle. If the particle starts from a point where angular distance is α from the highest point

of the circle, show that it will fly off the curve when $3 \cos \theta = 2 \cos \alpha$. 4½

(*b*) A heavy particle hangs from a point O by a string of length *a*. It is projected horizontally with a velocity *v* such that $v^2 = \left(2 + \sqrt{3}\right) ag$, show that the string becomes slack when it has described as angle given by,

$$\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right).$$ 4½

(*c*) When an automobile moving with a speed of 36 km/h reaches an upward inclined road of angle 30°, its engine is switched off. If the coefficient of friction is 0.1, how much distance will the automobile move before coming to rest? Take $g = 10\ ms^{-2}$. 4½

6. (*a*) Find an expression for the centre of pressure of a triangular lamina whose angular points are at depths α, β and γ from the effective surface. Hence, find the depth of centre of pressure of a triangle with its vertex in the effective surface and its base horizontal. 5

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(*b*) Equal volumes of three fluids of different densities, which do not mix together completely, fill a circular tube which is kept in a vertical plane. Prove that, if the densities of the fluids are in A.P., the common surface of the lightest and the heaviest fluids is at the extremity on the horizontal diameter of the circle. 5

(*c*) A parallelogram has the highest angular point in the surface of the liquid and one diagonal horizontal. Show that the depth of its centre of pressure is $\frac{7}{12}$ of the depth of the lowest point. 5