[This question paper contains 3 printed pages.]

4708 *Your Roll No.* .....

B.Sc. (Gen)/III **AS**

MATHEMATICAL SCIENCES (STATISTICS) – Paper V

Statistical Inference

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **five** *questions.*
All *questions carry equal marks.*

1. (a) Show that $\dfrac{\left[\sum_{i=1}^{n} x_i \left(\sum_{i=1}^{n} x_i - 1\right)\right]}{n(n-1)}$ is an unbiased estimate of θ^2, for the sample $x_1, \ldots, x_n$ drawn on X which takes the values 1 or 0 with respective probabilities θ and $(1-\theta)$.

   (b) Let $X \sim U(0, \theta)$ and $X_1, \ldots, X_n$ be a random sample of n observations of X. Show that $2\overline{X}$ is a consistent estimator of θ.

2. (a) Show that the sample mean $\overline{X}$ is sufficient for estimating the parameter θ of the Poisson distribution.

P.T.O.

(b) For a random sample of size n from $N(\mu, \sigma^2)$, find the MLE for μ and σ^2.

3. (a) Show that $\bar{X}$ in random sampling from

$$f_\theta(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & \text{if } 0 < x < \infty \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < \theta < \infty$, is an MVB estimator of θ.

(b) Let T_1 and T_2 be two unbiased estimators of $\Gamma(\theta)$ having the same variance. Show that their correlation coefficient P_θ cannot be smaller than $2e_\theta - 1$, where e_θ is the efficiency of each estimator.

4. (a) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = 1/2$ against $H_1 : p = 3/4$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of Type I error and power of the test.

(b) Construct SPRT for testing $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$ if X is poisson with parameter λ and $\alpha = \beta = 0.1$.

5. (a) Explain in brief the concept of interval estimation.

(b) Explain the main differences between the parametric and non-parametric approaches to the theory of statistical inference.

6. (a) Explain the following terms :

(i) Errors of first and second kind

(ii) The best critical region

(iii) Simple and composite hypotheses

(iv) Parametric space and sample space

(b) Examine whether the MLEs of the parameters are identical with those obtained by method of moments in random sampling from the distribution

$$f\left(x, \mu, \sigma^2\right) \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad -\infty < x < \infty .$$

7. Write short notes on any **two** of the following :

(i) Statement of Cramer-Rao inequality along with the regularity conditions, and its significance in point estimation.

(ii) Method of Minimum Chi-square.

(iii) Method of Maximum Likelihood.