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Your Roll No. ....

8063

B.Sc. (G)/III

D

MATHEMATICS—Paper VI (ii)

(Statistics)

Time : 3 Hours

Maximum Marks : 55

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any two parts from each question.

Third question carries ten marks and

others carry nine marks each.

1. (a) The first four moments of a distribution about the value 4 of a variable are  $-1.5$ ,  $17$ ,  $-30$  and  $108$ . Find the moments about mean. 4½
  
- (b) Prove that the sum of the squares of the deviations of a set of values is minimum when deviations are measured from the mean. 4½

P.T.O.

- (c) Find the mean deviation from the mean and the standard deviation of the A.P. 4½

$$a, a + d, a + 2d, \dots, a + 2nd.$$

2. (a) A random variable X has the following probability function :

X	P(x)
0	0
1	$k$
2	$2k$
3	$2k$
4	$3k$
5	$k^2$
6	$2k^2$
7	$7k^2+k$

- (i) Find  $k$ , and

- (ii) Evaluate  $P(X < 6)$ ,  $P(X \geq 6)$ .

- (b) Obtain the mean and variance of :

$$Y = 2X_1 + 3X_2 + 4X_3$$

where  $X_1$ ,  $X_2$  and  $X_3$  are three random variables with means given by 3, 4, 5 respectively and variances by 10, 20 and 30 respectively, and covariances by :

$$\sigma_{12} = 0, \sigma_{23} = 0, \sigma_{13} = 5,$$

where  $\sigma_{ij} = \text{cov}(X_i, X_j)$ . 4½

- (c) Let the random variable  $X$  assume the value  $r$  with the probability law :

$$P(X = r) = q^{r-1} p; r = 1, 2, 3 \dots$$

Find the m.g.f. and hence mean and variance. 4½

3. (a) In eight throws of a die 5 or 6 is considered a success. Find the mean and the standard deviation of the number of successes. 5

(b) If  $X$  follows  $B(n, p)$ , show that :

5

$$\text{cov}\left(\frac{X}{n}, \frac{n-X}{n}\right) = -\frac{pq}{n}.$$

(c) If  $X$  is a Poisson variate such that :

$$P(X = 2) = 9 P(X = 4) + 90 P(X = 6).$$

Find the mean of  $X$ .

5

4. (a) For a continuous distribution whose probability density function is given by :

4½

$$f(x) = \frac{3}{4}x(2-x), \text{ for } 0 \leq x \leq 2,$$

(i) Find the mean deviation about mean, and

(ii) Find the  $(2n + 1)$ th moment  $\mu_{2n+1}$  about mean for each positive integer  $n$ .

- (b) In a certain examination test 2000 students appeared in mathematics, the average marks obtained were 50% and the standard deviation was 5%. How many students do you expect to obtain more than 60% marks where marks are assumed to be distributed normally ? (Given that the area under the standard normal curve between  $z = 0$  and  $z = 2$  is 0.4772).

4½

- (c) Let  $X$  and  $Y$  be independent normal variates with means 6, 7 and variances, 9, 16 respectively. Determine the value of  $\lambda$  such that :

4½

$$P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda).$$

5. (a) Fit a straight line to the following data taking  $y$  as the dependent variable :

4½

$x$	$y$
0	12
5	15

10	17
15	22
20	24
25	30

- (b) If  $x$  and  $y$  are two correlated variables with the same standard deviation and the correlation coefficient  $r$ , show that the correlation coefficient between  $x$  and  $x + y$  is :  $4\frac{1}{2}$

$$\sqrt{\frac{1+r}{2}}$$

- (c) Two random variables have the least squares regression lines :  $4\frac{1}{2}$

$$3x + 2y = 26 \text{ and}$$

$$6x + y = 31$$

Find the mean values and the correlation coefficient.

6. (a) A sample of 900 days is taken from metrological records of a city and 100 of them are found to be foggy. What are the probable limits to the percentage of foggy days in the city ?  $4\frac{1}{2}$

(b) Two independent samples of 8 and 7 items respectively

had the following values :

4½

$x$	$y$
9	10
11	12
13	10
11	14
15	9
9	8
12	10
14	

Do the two estimates of population variance differ significantly ? (Given that for (7, 6) d.f. the value of F at 5% level of significance is 4.21).

- (c) A die is thrown 120 times and each outcome is recorded as under :

Faces	Frequency
1	20
2	22
3	17
4	18
5	19
6	24

Show that the die is unbiased. (Given that  $\chi^2$  for 5 degrees of freedom at 5% level is 11.07). 4½