This question paper contains **4** printed pages]

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B.Sc.(Gen.)/III   **D**

MATHEMATICAL SCIENCES (STATISTICS)—Paper V

Statistical Inference

Time : 3 Hours   *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *five* questions.

All questions carry equal marks.

1.   (*a*)   Show that an MVU estimator is unique, in the sense that if both T_0 and T_1 are MVU estimators for $\gamma(\theta)$, then $T_0 = T_1$ almost everywhere for each $\theta \in \Theta$.

     (*b*)   Explain the following terms :

         (*i*)   Most Powerful Critical Region

         (*ii*)   Type I and Type II errors

         (*iii*)   Simple and Composite hypothesis.

2.   (a)   A random variable X takes values 0, 1, 2 with respective

probabilities :

$$\frac{\theta}{4N} + \frac{1}{2}\left(1 - \frac{\theta}{N}\right), \frac{\theta}{2N} + \frac{\alpha}{2}\left(1 - \frac{\theta}{N}\right), \text{ and}$$

$$\frac{\theta}{4N} + \frac{1-\alpha}{2}\left(1 - \frac{\theta}{N}\right),$$

where N is a known number and α, θ are unknown

parameters. If 75 independent observations on X

yielded the values 0, 1, 2 with frequencies 27, 38, 10

respectively ($\Sigma f_i = 75$), estimate θ and α by the method

of moments.

(b)   Show with the help of suitable examples that Maximum

Likelihood Estimators are not in general unique and

unbiased.

3. (a) State and prove Rao-Blackwell theorem.

    (b) Let p be the probability that a coin will fall head in a single toss in order to test $H_0 : p = \dfrac{1}{2}$ against $H_1 : p = \dfrac{3}{4}$. The coin is tossed 6 times and H_0 is rejected if more than 4 heads are obtained . Find the probability of Type I error and power of the test.

4. (a) Show that the sample mean $\bar{X}$ is sufficient for estimating the parameter λ of the Poisson distribution.

    (b) The ABC company claims that the lifetime of a type of battery that it manufactures is more than 250 hours. A consumer advocate, wishing to determine whether the claim is justified, measures the lifetimes of 24 of the company's batteries; the results are listed in the table below. Assuming a significance level of 0.05, determine

whether the sample lifetimes of the batteries produced

by the ABC company are random.

271	230	198	275	282	225	284	219
253	216	262	288	236	291	253	224
264	295	211	252	294	243	272	268

5. Let $X_{(1)}$, $X_{(2)}$, $X_{(3)}$ be the ordered statistics of a random

sample of size 3 drawn from $f(x, \theta) = \dfrac{1}{\theta}$, $0 < x < \theta$, where

$\theta > 0$. Show that $4X_{(1)}$, $2X_{(2)}$ and $\dfrac{4}{3}X_{(3)}$ are all unbiased

estimator for θ. Find the variance and hence the efficiency

of each estimator.

6. Write short notes on the following :

(a) Wald's Sequential Probability Ratio Test

(b) Statement and the Regularity conditions of the Cramer-
Rao inequality

(c) Median test in the case of non-parametric inference.