

*This question paper contains 4 printed pages.*

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Your Roll No. ....

**B.Sc. (G) / III**

**D**

**MATHEMATICAL SCIENCES**

**(OPERATIONAL RESEARCH)**

**Paper V — Queueing Theory and Reliability**

*Time : 3 hours*

*Maximum Marks : 55*

*(Write your Roll No. on the top immediately  
on receipt of this question paper)*

*Answer any five questions. All questions carry equal marks.*

1. (a) Define a queueing system. Differentiate between the following:
  - (i) Steady state and transient solution
  - (ii) State dependent and state independent queueing system
  - (iii) Deterministic and probabilistic queueing system. 2+1+1+1
- (b) Define the reliability and hazard rate of a system. Find the reliability, failure rate and MTBF of a

P.T.O.

system if its failure time density function is exponential. 2+2+1+1

2. Derive the steady state probability distribution of the number of units in the generalized birth-death queueing model and discuss the queue with discouraged arrival as a particular case. 8+3
3. What is UTR? Derive the inherent availability for series system with constant failure rate and constant repair rate. 2+9
4. (a) Explain the difference between age replacement, corrective maintenance and preventive maintenance policy. 5  
  
(b) In a certain bank, customers arrive according to a Poisson distribution with a mean of 10 per hour. From observations on the teller's performance, the mean service time is estimated to be 4 minutes, with a variance of 8 (minutes)<sup>2</sup>. It is felt that the Erlang would be a reasonable assumption for the distribution of the teller's service time. Also it is assumed that there is no limit on the number of customers joining the queue. On the average, how

long must a customer wait before service, and how many customers are waiting for service? 3+3

5. (a) Write the differential-difference equations of the number of stages in an  $E_k | M | I | \infty$  : FCFS queueing system. 3

(b) Derive the reliability of a 2-unit parallel system with identical components under the assumption that both failure and repair rates are constant. 8

6. (a) The branch of Canara Bank has only one typist. The typing rate is randomly distributed approximating a Poisson distribution with mean service rate of 8 letters per hour. The letter arrives at a rate of 5 per hour during 8-hour work day.

Determine:

(i) Average number of letters in the system

(ii) Traffic Intensity or the utilization of typewriter and

(iii) The probability that the typist is idle. 2+1+1

(b) Discuss the optimal replacement policy of a system when the system deteriorates gradually and the value of money does not change with time. 7

7. (a) Define a series-parallel system. Find the failure rate and MTBF of such a system under the assumption that all the components are identical and have exponential failure time density function. 7
- (b) Determine the reliability of a two unit standby system when the failure time of each component follows a negative exponential distribution. 4
8. (a) For the queueing model  $D | D | 1 | K-1$ , obtain  $n(t)$ , the number of units in the system at time  $t$ , and  $W_q^{(n)}$ , the waiting time of the  $n$ th arrival before his service starts. Assume that, initially, the system is empty and service time is a multiple of inter arrival time. If  $\frac{1}{\lambda} = 2$ ,  $\frac{1}{\mu} = 6$  and  $K=8$ , find:
- (i) time where the first balk occurs
- (ii)  $n(10)$
- (iii)  $W_q^{(7)}$  5+1+1+1
- (b) Explain bulk queueing system with example. 3