[This question paper contains 4 printed pages.]

1875 *Your Roll No.* .....

B.Sc. (G) / III **E**

MATHEMATICAL SCIENCES

(OPERATIONAL RESEARCH)

Paper V – Queueing Theory and Reliability

Time : 3 hours *Maximum Marks : 55*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer any **five** *questions.*
All *questions carry equal marks.*
Simple calculators are allowed.

1. (a) Define queueing system. What are the different characteristics of a queueing system ? Explain the following :

   (i) Steady state and transient state solution

   (ii) State dependent and state independent system (2+1+1+1)

   (b) How a reliability of system differs from availability ? Find the reliability, failure rate and MTBF of a system if its failure time density function is exponential. (2+2+1+1)

P.T.O.

2. Derive the steady state probability solution of the number of units in the M/M/C: ∞/FCFS queueing system. Also derive the expression for the expected number of units in the queue for the same system. (7+4)

3. What is UTR ? Derive the inherent availability for series system with constant failure and repair rate. (2+9)

4. (a) Explain the difference between age replacement, corrective maintenance and preventive maintenance. (5)

(b) Consider a self-service store with one cashier. Assume Poisson arrivals and exponential service times. Suppose that on average nine customers arrive every 5 minutes and that the cashier can serve 10 in 5 minutes. Find

(i) average number of customers queue for service

(ii) probability of having more than 10 customers in the system and probability that a customer has to wait more than 2 minutes. (3+3)

5. (a) Write the differential-difference equations for the probabilities of having 'n' stages in an $M/E_k/1$: ∞/FCFS queueing system. (4)

(b) Derive the reliability of a 2-unit standby system with identical components under the assumption that both failure and repair rate are constant. (7)

6. (a) In a certain bank, customers arrive according to a Poisson distribution with a mean of 10 per hour. From observations on the teller's performance, the mean service time is estimated to be 4 minutes, with a variance of 8 (minutes)^2. It is felt that the Erlang would be a reasonable assumption for the distribution of the teller's service time. Also it is assumed that there is no limit on the number of customers joining the queue. Bank officials wish to know, on the average, how long a customer must wait before service, and how many customers are waiting for service. (3+3)

(b) Discuss the replacement policy of a system when the value of money does not change with time. (5)

7. Define a series-parallel system. Find the failure rate and MTBF of such a system under the assumption that all the components have the same reliability function. (3+8)

8. (a) For the queueing model D/D/1/K-1, obtain n(t), the number of units in the system at time t. Assume that, initially, the system is empty and service time is a multiple of inter arrival time. (4)

(b) Explain bulk queueing system. Give some examples of bulk arrivals. Also derive the differential-difference equations for the probabilities of having 'n' in an $M^x/M/1$ queueing system. (2+2+3)