[This question paper contains 4 printed pages.]

**1881** *Your Roll No.* .............

**B.Sc. (G) / III / NS** **E**

MATHEMATICS – Paper V

(Real Analysis)

*Time : 3 Hours* *Maximum Marks : 55*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Attempt any* **two** *parts from each question.* **Third** *question carries* **ten** *marks and others carry* **nine** *marks each.*

1. (a) Prove that the set of all rational numbers is not order complete.

(b) Define an open sets. Show that the union of an arbitrary family of open sets is open. Is the union of an arbitrary family of closed sets is closed? Justify your answer.

(c) Define a bounded set of real numbers and prove that an infinite bounded set of real numbers has a limit point. (4½,4½,4½)

2. (a) If $\langle a_n \rangle$, $\langle b_n \rangle$ are sequences of real numbers such that $\log_{n\to\infty} a_n = a$, $\log_{n\to\infty} b_n = b$, then prove that $\log_{n\to\infty}(a_n + b_n) = a + b$.

(b) Show that a bounded sequence converges to a number $l$ if and only if $\lim\sup a_n = \lim\inf a_n = l$.

(c) Show that the sequence $\langle a_n \rangle$ defined by

$$a_n = \frac{1}{2}\left(a_{n-1} + a_{n-2}\right),\ n \geq 2,$$

then show that converges. What is limit of $\langle a_n \rangle$ ?

(4½,4½,4½)

3. (a) State and prove Cauchy's nth root Test.

(b) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1.3.5....(2n-1)}{2.4.6.....(2n)} x^n$$

for all positive values of x.

(c) Test for convergence and absolute convergence for the following series :

$$1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \ldots$$ (5,5,5)

4. (a) State and prove intermediate value theorem.

(b) Let f be continuous on the closed interval [a,b], prove that there exist a real number M such that $f(x) \le M$, for all $x \in [a, b]$.

(c) Define uniformly continuity of a function f defined on an interval. Show that the function $f(x) = \sin x$, $x \in (0, \infty)$ is uniformly continuous on $(0, \infty)$. (4½,4½,4½)

5. (a) State and prove Rolle's theorem. Also give geometrical interpretation of the theorem.

(b) Prove that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0, 1].

(c) Use Taylor's theorem to prove that

$$1 + x + \frac{x^2}{2} < e^x < 1 + x + \frac{x^2}{2}e^x$$

for all $x > 0$. (4½,4½,4½)

6. (a) Find the maximum and minimum value of the function $f(x) = x^4 + 4x^3 - 2x^2 - 12x + 7$, $x \in R$.

(b) Prove that

$$\cos x \ge 1 - \frac{x^2}{2}, x \in R$$

(c) Evaluate the following :

(i) $\lim_{x\to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$

(ii) $\lim_{x\to 0} \left(\dfrac{1}{x}\right)^{\tan x}$ (4½,4½,4½)

(100)