1882

[This question paper contains 3 printed pages.]

Roll No. ....

Name of the Course: B.Sc. (G) Mathematical Sciences

Title of the paper : Mechanics - Paper VI (i) E

Annual Mode : III Year

Time: 3 Hours **Maximum Marks: 55**

(Write your Roll No. On the top immediately on receipt of this question paper)

Attempt any **TWO** parts from each question.

All questions are compulsory.

1. (a) A light rigid rod of length 2b terminated by heavy particles of weight 'w' and 'W' is placed inside a smooth hemispherical bowl of radius 'a' which is fixed with its plane horizontal. If particles of weight 'w' rests just below the plane of the bowl, prove that $wa^2 = W(2b^2 - a^2)$. (4½)

   (b) A particle is in equilibrium under the action of six forces. Three of these forces are reversed and the particle remains in equilibrium. Prove that it will still be in equilibrium if these forces are removed altogether. (4½)

   (c) Three forces P,Q,R act along the sides of a triangle formed by the lines $x + y = 1$, $y - x = 1$ and $y = 2$. Find the equation of the line of action of the resultant. (4½)

2. (a) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined at their extremities so as to form a hexagon. The rod AB is fixed in a horizontal position and the middle joints of AB and DE are joined by a string. Prove that the tension in the string is 3W. (4½)

   (b) Find the mass centre of a cubical box with no lid and the bottoms being made of the same thin material. (4½)

(c) A heavy uniform rod of length 2a rests with its ends in contact with two smooth inclined planes of inclinations 'α' and 'β' with the horizontal. The rod is in a vertical plane perpendicular to the common edge. If 'θ' is the inclination of the rod to the horizontal, prove that $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$. (4½)

3. (a) A pendulum is carried to the top of a mountain half a mile high. By how much must its present length be shortened so that it may beat seconds at the top of the mountain (4½)

(b) A particle of mass 'm' moves on a straight line under the influence of a force directed towards the origin 'O' on the line and proportional to the distance from O. The force at a unit distance is of magnitude mk^2. The particle passes O with velocity 'u', if x be its co-ordinate at time t and v be its velocity at that instant, show that

$$v^2 = u^2 - k^2x^2.$$ (4½)

(c) If a pendulum of length l makes 'n' complete oscillations in a given time, show that if the length be changed to l to l + l', the number of oscillations lost is n l' / 2 l. (4½)

4. (a) A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection . Prove that the angle of projection is

$$\tan^{-1}\frac{bc}{a(c-a)}$$ (4½)

(b) Find the Centre of Pressure of rectangular area with a side in effective surface (4½)

(c) A particle is projected under gravity with velocity $\sqrt{2bg}$ from a point at a height h above a level plane . Show that the angle of projection β for a maximum range along the plane is given by

$$\tan^2\beta = \frac{b}{(b+h)}$$ (4½)

5. (a) Equal volumes of three fluids of different densities, which do not mix together completely fill a circular tube which is kept in a vertical plane. Prove that if the densities of the fluid are in A.P. the common surface of the lightest and heaviest fluids is at an extremity on the horizontal diameter of the circle . (5)

(b) If there be 'n' liquids arranged in strata of equal thickness 'h' and the density of the upper most being , ρ that of the next 2ρ and so on that of the lowest being $n\rho$. Prove that the pressure at the lowest stratum is $[\frac{1}{2} n (n+1) g \rho h]$. (5)

(c) Show that the depth of centre of pressure of a trapezium of which one side of length 'a' is in the surface and the parallel side of length 'b' is at depth 'h' is

$$[(a + 3b) /(a + 2b)].h/2.$$ (5)

6. (a) Find the potential energy of a particle attracted towards a fixed point by a force of magnitude k^2/r^n , r being the distance from the fixed point and k, n any constants. (4½)

(b) A quadrilateral is immersed vertically having two sides of lengths 2a, a, parallel to the surface at depths h, 2h respectively . Show that the depth of the centre of pressure is $3h/2$. (4½)

(c) A small stone of mass 'm' is thrown vertically upward with initial velocity 'v'. If the air resistance at speed 'v' is mkv^2, where 'k' is the constant. Find the time from the ground level to the highest point. (4½)