[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 5023 **D** **Your Roll No...............**

Unique Paper Code : 235566

Name of the Course : **B.Sc. Physical Sciences / B.Sc. Mathematical Sciences**

Name of the Paper : MAPT–505 : Mathematics – V, Real Analysis

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Each part of question 1 is compulsory.
3. Attempt any **two** parts from each of the remaining questions.

1. (a) Using the fact that the countable union of countable sets is countable, prove that N × N is a countable set.

   (b) Find Inf(bS) and b Inf(S), where $b = -2$, $S = \{x : |x| < 1\}$.

   (c) Define limit point of a sequence. Find limit points of the sequence $\{(-1)^n + 3\}$.

   (d) Verify Bolzano Weierstrass theorem for the set $\{x + 2 : x \in (0, 1)\}$.

   (e) State Cauchy convergence criteria for the series.

   (f) Find the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{x^{(n-1)}}{n}$. (2½×6=15)

2. (a) Prove that the order completeness property fails for the set of rational numbers. (6)

(b) State Archimedean property of real numbers. Show that

$$\text{Sup}\left\{1-\frac{1}{n} : n \in N\right\} = 1.$$ (6)

(c) Find limit points of the following sets

(i) $\left\{\frac{1}{n} : n \in N\right\}$

(ii) Q, the set of rational numbers

(iii) $(0,1) \cup \left\{\frac{1}{n} : n \in N\right\} \cup \left\{2+\frac{1}{n} : n \in N\right\}$ (6)

3. (a) Show that the sequence $\langle r^n \rangle$ converges if $-1 < r \leq 1$. What happens if $r = -1$? (6)

(b) (i) State squeeze theorem and using this show that $\lim_{n\to\infty} \frac{\sin(n)}{n} = 0$.

(ii) Show that

$$\lim_{n\to\infty}\left[\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\frac{1}{(n+3)^2}+\ldots\ldots\ldots+\frac{1}{(n+n)^2}\right]=0$$ (6)

(c) Prove that the sequence $\langle a_n \rangle$ defined by

$$a_1 = 1,\ a_n = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+-----+\frac{1}{(n-1)!},\ (n \geq 2)$$

converges. (6)

4. (a) State Cauchy's n^{th} root test for an infinite series and hence test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n+1)^n x^n}{n^{(n+1)}}$, $x > 0$ (6)

(b) Test the convergence of the following series

(i) $\sum_{n=1}^{\infty}\left\{\sqrt[3]{n+1}-n\right\}$

(ii) $\sum_{n=1}^{\infty}\text{Cos}\left(\frac{1}{n^2}\right)$ (6)

(c) Define absolute and conditional convergence of an alternating series. Prove that the absolute convergence implies convergence but the converse is not true. (6)

5. (a) State and prove Ratio test for the convergence of a positive term series. (6)

(b) Determine the interval of convergence of the power series

$$\sum_{n=1}^{\infty}\frac{(-1)^{n+1}(x-1)^n}{n}$$ (6)

(c) Define sine function in terms of power series. Prove that

(i) $S(x+y) = S(x)C(y) + C(x)S(y)$

(ii) $C(x+y) = C(x)C(y) - S(x)S(y) \quad \forall\ x, y \in R$,

where C and S denote cosine and sine respectively. (6)

6. (a) State Weierstrass M – Test for the uniform convergence and test for uniform convergence the series $\sum_{n=1}^{\infty}\frac{\sin(x^2+nx^2)}{n(n+1)}$ for all real x. (6)

(b) Show that the sequence $\langle f_n \rangle$ of functions where $f_n(x) = x^n$, is uniformly convergent on [0, k], k < 1 and only pointwise convergent on [0, 1]. (6)

(c) Show that the sequence $\langle f_n \rangle$ of functions, where $f_n(x) = nxe^{-nx^2}$, $x \geq 0$ is not uniformly convergent in [0, k], k > 0. (6)