[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1569 **C** **Roll No.....**

Unique Paper Code : 235666

Name of the Course : B.Sc. Physical Sciences/Applied Physical Scienes

Name of the Paper : Mechanics and Discrete Mathematics : MAPT–606

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated.

1. (a) Two beads of weights w and w′ can slide on a smooth circular wire in a vertical plane. They are connected by a light string which subtends an angle 2β at the centre of the circle when the beads are in equilibrium on the upper half of the wire. Prove that the inclination α of the string to the horizontal is given by

$$\tan\alpha = \frac{w - w'}{w + w'}\tan\beta \qquad (8)$$

(b) Two light rings carl slide on a rough horizontal rod. The rings are connected by a light inextensible string of length a, to the midpoint of which is attached a weight w. Show the greatest distance between the rings, consistent with equilibrium of the system, is $\frac{\mu a}{\sqrt{1+\mu^2}}$, where μ is the coefficient of friction between either ring and the rod. (8)

P.T.O.

(c) Find the mass centre of a cubical box with no lid, the sides and bottom being made of the same thin material. (8)

2. (a) Prove that for a particle moving along a curve's the tangential and normal Components of a velocity are $(\dot{s},0)$ and of acceleration are $\left(\frac{dv}{dt}, \frac{v^2}{\rho}\right)$, where v is the speed and ρ is the radius of curvature of the curve at the point. (8)

(b) A gun is mounted on a hill of height h above a level plain. Show that if the resistance of air is neglected, the greatest horizontal range for given muzzle velocity v is obtained by firing at an angle of elevation θ such that

$$\operatorname{cosec}^2\theta = 2\left(1 + \frac{gh}{v^2}\right). \quad (8)$$

(c) (i) A uniform accelerated automobile passes two telephone poles with velocities 10 and 20 mph, Calculate its velocity when it is halfway between the poles. (4)

(ii) A particle of mass m moves on a straight line under the influence of a force directed towards the origin O on the line and proportional to the distance from O, the force at unit distance is of magnitude mk^2. The particle passes O with an initial velocity u. If x is the coordinate at time t and v its velocity at that instant, show that

$$v^2 + k^2x^2 = u^2. \quad (4)$$

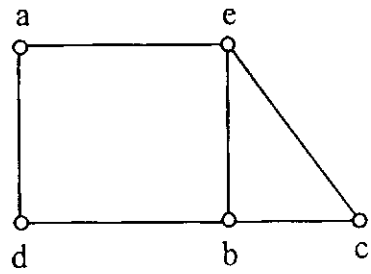
3. (a) (i) Define complete graph. Using principle of mathematical induction prove that number of edges in K_n is $\frac{n(n-1)}{2}$. (4)

(ii) Draw the graph K_6 (3)

(b) Define in-degree and out-degree of a vertex in a directed graph. Prove that if G = (V, E) is a directed graph, then $\sum_{x \in V} \deg^-(x) = \sum_{x \in V} \deg^+(x) = |E|$. (7)

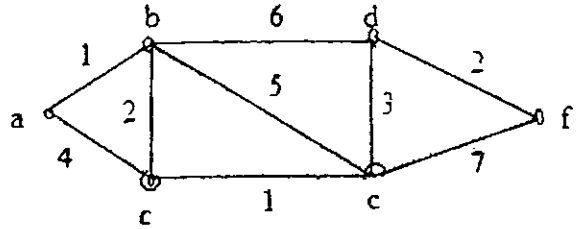
(c) Show that if G is a bipartite simple graph with n vertices and e edges, then $e \leq \frac{n^2}{4}$. (7)

4. (a) How many paths of length four are there from a to e in the graph G ? Identify all the paths.



G (7)

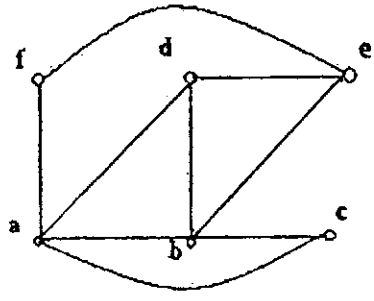
(b) Define weighted graph. Use Dijkstra's algorithm to find the length of a shortest path between a and f in the following weighted graph



(7)

(c) Prove that a tree with n vertices has n – 1 edges.

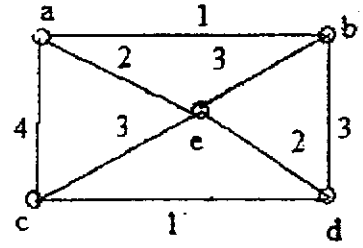
5. (a) Use depth-first search to find a spanning tree for the given graph G (7)



G (7½)

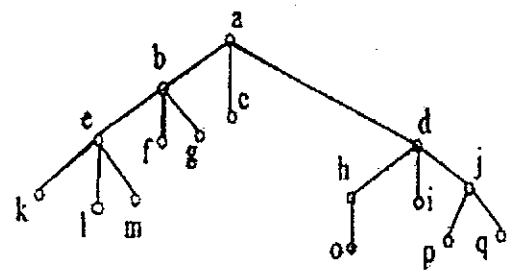
P.T.O.

(b) Use kruskal's algorithm to find a minimum spanning tree in the following weighted graph



(7½)

(c) Determine the preorder, postorder traversal of the given rooted tree T.



T

(7½)