Sr. No. of Question Paper : 7006      **D**      Your Roll No................

Unique Paper Code    : 235267

Name of the Course    : **B.Sc. Applied Physical Sciences (Analytical Chemistry) (Part I)**

**B.Sc. Applied Physical Sciences (Industrial Chemistry) (Part I)**

Name of the Paper    : MAPT-101 : Calculus and Matrices

Semester    : II

Duration : 3 Hours             Maximum Marks : 75

## Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** questions from each Section.

3. Use of non-programmable scientific calculator is permitted.

## SECTION I

1. (a) Determine whether the system of equations given below is consistent or not and solve, if the system is consistent.

    $x - 4y + 7z = 8$

    $3x + 8y - 2z = 6$

    $7x - 8y + 26z = 31$

    (b) Is set $A = \{(x, y, z)$ where $x, y, z \in R$ and $x = y + z\}$ a subspace of $R^3$ over R ? If yes, describe it geometrically.

    (c) Show that $Q = \{(1, -2), (2, 0)\}$ is a basis of $R^2$ over R.      (4,4,4)

2. (a) Determine any two distinct basis for R over R.

(b) Find inverse of matrix A, using elementary row operations, where

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}.$$

(c) Let $T : R^3 \to R^2$ be given by $T(x, y, z) = \begin{bmatrix} x+y \\ z-3 \end{bmatrix}$. Is T linear ?    (4,4,4)

3. (a) Using elementary row operations, reduce the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 0 & -3 \\ 3 & 3 & -3 \end{bmatrix}$ to triangular

form and hence determine the rank.

(b) Determine the linear transformation $T : R^2 \to R^2$ which represents reflection about x-axis. Also, find the matrix of reflection about x-axis.

(c) Find the characteristic equation, characteristic value and corresponding characteristic vectors for the matrix $\begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$.    (4,4,4)

## SECTION II

4. (a) Sketch the graph of $y = 2 - |x - 1|$. Mention the transformation used at each step.

(b) An amount of 10 ml of a medicine is injected into a patient's body. Half the amount of medicine is absorbed by the patient's body in 10 hours. How long will it take for the patient to absorb 70% of the medicine ?

(c) Draw the level curve of $f(x, y) = x^2 + y^2$ of height k = 1, 4.    (6,6,6)

5. (a) If $y = e^{m\cos^{-1}(x)}$, show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0$$

(b) Show that $z = e^x \sin(y) + e^y \cos(x)$ is a solution of Laplace's equation.

(c) Find Taylor series generated by $f(x) = \cos(x)$ about $x = 0$ (assuming the possibility of its expansion). (6,6,6)

6. (a) Discuss the convergence of the sequences :

(i) $\left\langle \dfrac{(-1)^n}{n^2} \right\rangle$

(ii) $\left\langle 10 + \dfrac{\cos^2(n)}{n^3} \right\rangle$

(b) If $z = 3xy - y^3 + (y^2 - 2x)^{3/2}$. Show that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}.$$

(c) Find $\dfrac{d^n y}{dx^n}$ where $y = \cos(2x - 3)$. (6,6,6)

## SECTION III

7. (a) Find equation with lowest possible degree with real coefficients having $2 + \sqrt{3}$ and $1 + i$ as its roots.

(b) Using DeMoivre's Theorem, solve $z^4 + z^3 + z^2 + z + 1 = 0$. (3.5,4)

8. (a) Are points whose affixes are represented by $1 + i$, $1 - i$, $3 - 3i$ collinear.

(b) Show that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} \cos\left(\dfrac{n\pi}{4}\right)$. (3.5,4)

9.   (a)   Find equation of circle described on the join of the points $1 + i$, and $1 - i$ as extremities of one of its diameters.

      (b)   Using DeMoivre's Theorem, find $\left(\sqrt{3}+i\right)^{1/3}$.                 (3.5,4)