[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1519 **E** **Your Roll No...............**

Unique Paper Code : 235267

Name of the Course : **B.Sc. Applied Physical Sciences (Industrial Chemistry)**

Name of the Paper : Calculus and Matrices : MAPT–101

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** questions from each section.
3. Use of scientific calculator is not permitted.
4. Marks are indicated.

SECTION I

1. (a) For what values of λ does the following system of equations have a solution ?

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

Also find the solution in each case.

(b) Find the characteristic values and corresponding characteristic vectors for the matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$ (6,6)

2. (a) Verify that the set $S = \{(0,1,0), (1,0,1), (1,1,0)\}$ of vectors is a basis of R^3.

(b) Let $T: R^3 \rightarrow R^2$ be a linear transformation such that $T(1,0,1) = (2,-1)$, $T(0,1,1) = (1,1)$, $T(1,1,0) = (-1,4)$. Find $T(1,1,1)$. (6,6)

3. (a) Find the inverse of the matrix A, using elementary row operations, where

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 1 & 2 \\ 0 & 1 & -1 \end{bmatrix}.$$

(b) Examine which of the following is a subspace of R^2. If it is a subspace, give its geometric representation :

(i) $V_1 = \{(a,b^2) : a, b \in R\}$.

(ii) $V_2 = \{(a, 2a) : a \in R\}$. (6,6)

SECTION II

4. (a) Radium is known to decay at a rate proportional to the amount present. If half of radium is 1600 years. What percentage of radium will remain in a given sample after 800 years ?

(b) Sketch the graph of functions (i) $y = \frac{1}{2}x^2 - 3x + \frac{11}{2}$, (ii) $y = -2x^2 - 4x - 4$.

Mention the transformation used at each step.

(c) Find the nth derivative of $y = x(x+1)\log(x+1)^3$. (6,6,6)

5. (a) If $y = e^{m\sin^{-1}x}$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.

(b) Discuss the convergence of the sequences :

(i) $\left\langle \frac{(-1)^n}{n^3} \right\rangle$ (ii) $\left\langle \frac{n^2}{n+3} \right\rangle$

(c) If u = f(r), where $r = \sqrt{x^2 + y^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$. (6,6,6)

6. (a) Find Taylor's series generated by $f(x) = \frac{1}{x}$ at x = 2, when does this series converge to $\frac{1}{x}$.

(b) Draw the level curves of height k = 1,2,5 for the surface $f(x,y) = 9x^2 + 25y^2$.

(c) Show that $U = \frac{1}{\sqrt{x^2 + y^2 + y^2}}$, $x^2 + y^2 + z^2 \neq 0$ is a solution of 3-dimentional Laplace equation. (6,6,6)

SECTION III

7. (a) Give the geometrical representation of product of two complex numbers.

(b) Prove that for any two complex number z_1 and z_2 :

$|z_1 - z_2| \geq ||z_1| - |z_2||$ (3.5,4)

8. (a) Prove that the points with affixes z_1, z_2 and z_3 are collinear if and only if

$$z_1(\bar{z}_2 - \bar{z}_3) + z_2(\bar{z}_3 - \bar{z}_1) + z_3(\bar{z}_1 - \bar{z}_2) = 0$$

(b) Use the De Moivre's theorem to solve the equation $z^4 + 1 = 0$. (3.5,4)

9. (a) Form an equation in lowest degree with real coefficients which has $2 - 3i$ and $3 + 2i$ as two of its roots.

(b) Find the all values of $\left(\sqrt{3}+i\right)^{1/3}$. (3.5,4)