This question paper contains 3 printed pages.

5184–F

Your Roll No. .....

B.Sc. Prog. / Sem. I

B

MATHEMATICAL SCIENCES

Paper OR-1 – Operational Research – I

(For Admissions of 2011 and onwards)

Time : 3 hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions.

1(a) Define Operations Research as a decision making science? What are the essential characteristics of Operations Research approach? **7.5**

(b) Prove the following

(i) If the convex set of feasible solutions of the system of equations $Ax = b$, $x \geq 0$, is a convex polyhedron, then at least one of the extreme points gives an optimal solution.

(ii) If the optimal solution occurs at more than one extreme point, then the value of the objective function will be the same for all convex combinations of these extreme points. **7.5**

2.(a) Wild West produces two types of cowboy hats. A type 1 hat requires twice as much labor time as type 2 hat. If all the available labor time is dedicated to produce type 2 hats, a total of 400 can be produced each day. The respective market limits for the two types are 150 and 200 hats per day. The profit is Rs. 8 per type 1 hat and Rs. 5 per type 2 hat. Determine the number of hats of each type that would maximize profit. Formulate and solve this problem as a linear programming problem. (Use Graphical Method.) **7·5**

(b) Define a convex set and examine the convexity of the set: **7.5**

$$s = \{(x, y): 2x + 5y \geq 10\}.$$

3(a) Find the Initial basic feasible solution in the following transportation problem by Vogel's approximation. Also obtain the optimal solution. Is there an alternative optimal solution? If yes, then determine one of them.

Turn over

	Destinations			
Sources	**D1**	**D2**	**D3**	**Supply**
S1	4	8	8	76
S2	16	24	16	82
S3	8	16	24	77
Demand	72	102	41	

7.5

(b) Use 2-phase Simplex Method to solve the following LPP

$$\text{Max } Z = 5x_1 + 8x_2$$
$$\text{subject to } 3x_1 + 2x_2 \geq 3$$
$$x_1 + 4x_2 \geq 4$$
$$x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0.$$

7.5

4(a) A company is considering an expansion into five new sales territories. The company has recruited four new salesmen. Based on the salesman's experience and personality traits, the sales manager has assigned a rating to each of the sales territories. The ratings are as follows:

	Territory				
Salesman	**1**	**2**	**3**	**4**	**5**
A	**75**	**80**	**85**	**70**	**90**
B	**91**	**71**	**82**	**75**	**85**
C	**78**	**90**	**85**	**80**	**80**
D	**65**	**75**	**88**	**85**	**90**

Suggest optimal assignment of the salesmen, so as to minimize the total cost. **6**

(b) Define the following terms (i) Extreme point (ii) Basic feasible solution **4**

(c) Determine all possible basic feasible solutions for the following set of equations

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

5

5(a) What is an infeasible solution, and how does it occur?
How is this condition recognized in the graphical method? **4**

(b) Write the dual of the following linear programming problem

$$MinZ = 3x_1 + 4x_2 + x_3$$

subject to $-2x_1 + 3x_2 - x_3 \geq 1$

$x_2 + x_3 = 10$

$x_1 + 4x_2 + 7x_3 \leq 92$

x_1 *is unrestricted*, $x_2, x_3 \geq 0$ **5**

(c) State and prove Weak duality theorem. **6**

6(a) With the help of simplex method verify whether the following problem is inconsistent or unbounded:

$$MaxZ = 6x_1 + 4x_2$$

subject to $x_1 + x_2 \leq 5$

$x_2 \geq 8$

$x_1, x_2 \geq 0$ **7.5**

(b) Find the inverse of the matrix using simplex method **7.5**

$$\begin{pmatrix} 2 & 3 \\ 4 & 2 \end{pmatrix}$$

7 Write short note on any three of the following:

(a) Scope and limitations of the study of Operational Research.

(b) Scientific method approach of Operational Research.

(c) Degeneracy in linear programming.

(d) Solution of a system of linear equations using simplex method.

(e) Economic interpretation of duality in linear programming. **15**