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Your Roll No. .....

5184-H

B.Sc. (Life Science)/I Sem. **B**

Paper MACT-303

MATHEMATICS AND STATISTICS

(Admission of 2010 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are *three* Sections in this question paper.

Attempt any *two* questions from each Section.

Students are allowed to use simple calculator.

Section I

1. (*a*) If

$$y = (a + bx)\, e^{2x},$$

show that :

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0. \qquad 5\frac{1}{2}$$

P.T.O.

(*b*) The function $f(x)$ is defined as

$$f(x) = \begin{cases} x - 1 & , \quad x < 0 \\ \frac{1}{4} & , \quad x = 0. \\ x^2 & , \quad x > 0 \end{cases}$$

Discuss the continuity at $x = 0$. 5½

(*c*) Examine the convergence of the following sequence and find its limit if it exists :

$$\left\langle \frac{\sin n}{n} \right\rangle. \qquad 5½$$

2. (*a*) A population of size 50000 grows according to the rule $N = 50000 + 90t^2$, where the time t is measured in days. Find the average growth rate in the time interval $t = 0$ to $t = 2$. 5

(*b*) Evaluate :

$$\lim_{n \to \infty} \frac{3 + 5n^2}{n + n^3}. \qquad 3½$$

(*c*) Evaluate the following : 8

(*i*) $\int xe^x \, dx$

(*ii*) $\int x^2 \cos x \, dx$

(*iii*) $\int x^n \log x \, dx$

(*iv*) $\int \sec^2 4x \, dx.$

3. (*a*) Find $\frac{d^2y}{dx^2}$ if : 6

(*i*) $y = \tan^{-1} x$

(*ii*) $y = \left(\sqrt{x} + \frac{1}{x}\right)\left(x - \frac{1}{\sqrt{x}}\right).$

(*b*) Write down the first five terms of the sequence given by the recursion formula :

$$a_1 = 2, a_{n+1} = (-1)^{n+1} \frac{a_n}{2}.$$ 5

(*c*) Evaluate :

$$\int_0^1 \frac{3x^3 - 4x^2 + 1}{\sqrt{x}} \, dx.$$ 5½

4. (*a*) Ten rabbits numbered 1, 2, 3, ..., 10 are tested whether they react to a certain drug. We assign the number one if the reaction is positive and zero otherwise. Is this assignment a function ? Why ? Find the domain and range of this assignment. 5

(*b*) The size of a slowly growing bacteria culture is approximately given by

$$N(t) = N_0 + 52t + 2t^2$$

where N_0 is the size when $t = 0$ and t is the time in hours. Find the growth rate when time is 5 hours. 6½

(*c*) Sketch the graph of $y = \sin x$ in $[0, 2\pi]$. 5

Section II

5. (*a*) If

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 0 & 5 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 2 & 0 \\ 5 & 1 & 1 \end{bmatrix}$$

find $2A + 3B$. 5

(*b*) Find the values of x, such that :

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 4 & 8 \end{bmatrix} \begin{bmatrix} x \\ 6 \\ -1 \end{bmatrix} = 0.$$ 6

6. (*a*) Find the image of points (–1, 4), (2, –3) under reflection in X-axis as well as Y-axis by using matrix multiplication. 6

(*b*) For

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$$

verify that : $AB \neq BA$. 5

7. (*a*) Express the following a set of linear equations : 5

(*i*)
$$\begin{bmatrix} 3 & x & 0 \\ 4 & 1 & y \end{bmatrix} \begin{bmatrix} z & 0 \\ 0 & -3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 1 \end{bmatrix}$$

(*ii*)
$$\begin{bmatrix} 2 & -5 & 7 \\ 1 & -3 & 4 \\ 3 & -8 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \\ 11 \end{bmatrix}.$$

(*b*) If

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

and $$C = \begin{bmatrix} 3 & 1 & -2 \\ 0 & 1 & 4 \\ 1 & 0 & 5 \end{bmatrix}$$

prove that $A(B + C) = AB + AC$. 6

Section III

8. (*a*) The mean of 40 observations was 160. It was detected on rechecking that the value of 165 was wrongly copied as 125 for computations of mean. Find the correct mean. 5

(*b*) The incidence of occurrence of occupational disease in an industry is such that the workers have a 20% chance of suffering from it. What is the probablity that out of six workers four or more will catch the disease ? 5

9. (*a*) If X is a Poisson variate such that :

$$P(X = 1) = 2P(X = 2)$$

find mean and variance for X. 5

(*b*) Calculate the correlation coefficient of the following data : 5

X	Y
2	1
8	7
5	6
4	2
6	4

10. (*a*) If X is a normal variate with mean 10 and standard deviation 4. Determine the probability $P(X \geq 7)$.

$$\left[\text{Given : } \frac{1}{\sqrt{2\pi}} \int_0^{.75} e^{-z^2/2}\, dz = .2734\right] \quad 5$$

(*b*) Fit a straight line to the following data taking x as the independent and y as the dependent variable : 5

x	y
1	3
2	5
3	10
4	12
5	20

11. (*a*) The following table shows the age distribution of cases of a certain disease reported during a year :

Age	Number of Cases
5—15	5
15—20	10
20—25	20
25—30	22
30—35	13
35—40	5

Calculate the median for the above distribution. 5

(*b*) A coin is tossed three times. A man gets Rs. 5 for every head and loses Rs. 2 for every tail. Find his expectation. 5