

[This question paper contains 4 printed pages.]

5184-A

Your Roll No. ....

**B.Sc. (Prog.) Mathematical Sciences / I Sém. B**

Paper : STP-101 : Descriptive Statistics and Probability

(Admissions of 2011 and onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately  
on receipt of this question paper.)*

*Attempt any six questions.*

*All questions carry equal marks.*

1. (a) Describe how you can represent a frequency distribution diagrammatically using cumulative frequencies. How will you obtain the value of (i) median (ii) Ist quartile from this diagram.  
  
(b) Define Skewness and Kurtosis. How are they measured ? Find the limits for Bowley's coefficient of Skewness. (6,6½)
2. (a) In a frequency table, the upper boundary of each class interval has a constant ratio to the lower boundary. Show that the geometric mean  $G$  may be expressed by the formula :

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$$\log G = x_0 + \frac{C}{N} \sum_i f_i (i-1)$$

where  $x_0$  is the logarithm of the mid value of the first interval and  $C$  is the logarithm of the ratio between upper and lower boundaries.

(b) If  $r(X, Y) = 0$ , what is the value of  $\text{cov}(X, Y)$  and how are  $X$  and  $Y$  related?

(c) State the limits for rank correlation coefficient. When are these limits attained? (6,3,3½)

3. (a) For a random variable  $X$ , suppose that moments of all order exist. Denoting by  $\beta_K$ , the  $K$ th absolute moment show that

$$\beta_K^{2K} \leq \beta_{K-1}^K \beta_{K+1}^K$$

(b) Define 'dispersion'. Describe different measures of dispersion.

Let  $r$  be the range and  $S$  the standard deviation of a set of observations  $x_1, x_2, \dots, x_n$ , prove that  $S \leq r$ . (6,6½)

4. (a) What are regression lines? Why do we have two regression lines? Show that if one of the regression coefficients is greater than unity, the other must be less than unity.

- (b) Prove that Speerman's rank correlation coefficient ( $\rho$ ) is given by :

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}; \text{ where } d_i \text{ denotes}$$

the difference between the ranks of  $i$ th individual.

(6, 6½)

5. (a) Give the empirical (statistical) definition of probability and give its advantage over classical definition of probability.

- (b) Two-third of the students in a class are boys and the rest girls. It is known that the probability of a girl getting a first class is 0.25 and that of boy getting a first class is 0.28. Find the probability that a student chosen at random will get first class marks in the subject.

- (c) Given an Experiment such that  $P(A) = \frac{1}{2}$ ,

$$P(B) = \frac{1}{2}, P(A \cup B) = \frac{2}{3}. \text{ Find } P(\bar{A}), P(\bar{A} \cap \bar{B}),$$

$$P(A \cap B), P(\bar{A} \cup \bar{B}). \quad (2\frac{1}{2}, 6, 4)$$

6. (a) Four identical marbles marked 1, 2, 3 and 123 respectively are put in an urn and one marble is drawn at random. Let  $A_i$  ( $i = 1, 2, 3$ ) denote the event that the number  $i$  appears on the drawn marble. Discuss the independence of the events  $A_1$ ,  $A_2$  and  $A_3$ .

P.T.O.

- (b) A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelop just two consecutive letters TA are visible. What is the probability that letters came from TATANAGAR.  $(6\frac{1}{2}, 6)$

7. (a) Explain the principle of least squares. Use it to fit the curve  $Y = a \exp(bx + c)$  to a set of  $n$  points  $(X_i, Y_i)$ ,  $i = 1, 2, \dots, n$ .

- (b) If  $A$  and  $B$  are independent events, show that

(i)  $\bar{A}$  and  $B$  are independent

(ii)  $\bar{A}$  and  $\bar{B}$  are independent  $(6\frac{1}{2}, 6)$

8. (a) Two regression lines are  $X + 3Y - 5 = 0$  and  $4X + 3Y - 8 = 0$ . Find (i) regression coefficients  $b_{XY}$ ,  $b_{YX}$ . (ii) Correlation coefficient  $r$ .

- (b) Let  $A$  and  $B$  be events, neither of which has probability zero. Prove or disprove the following :

(i) If  $A$  and  $B$  are disjoint, then  $A$  and  $B$  are independent.

(ii) If  $A$  and  $B$  are independent, then  $A$  and  $B$  are disjoint.

- (c) Explain the symbols  $R_{123}$ ,  $r_{12.3}$  and  $r_{12}$ . How  $R_{1.23}$  and  $r_{12}$  are related?  $(6\frac{1}{2}, 4, 2)$