[This question paper contains 4 printed pages.]

Your Roll No. .....

5099 **B**

B.Sc. (Prog.)/I

MA - 107 (a) – MATHEMATICS – I

(For Physical Sciences)

(Admissions of 2008 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each Section.

SECTION – I

1. (a) A boat is travelling eastward across a river at the rate of 4 miles per hour while the river's current is flowing southward at a rate of 3 miles per hour. Find the resultant velocity of the boat. 6

   (b) Show that the set $\{(1,0,-1), (1,1,1), (1,2,4)\}$ is a basis for $\mathbb{R}^3$. 6

2. (a) Is $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ defined by

   $$T(x,y) = (0,\ x-y,\ x+y)$$

   a linear transformation ? 6

   (b) Find the eigen values and eigen vectors for the transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by

   $$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 3x \\ 3y \end{bmatrix}$$ 6

[P. T. O.

3. (a) Solve by elementary row operations, the following system of equations :

$$x + y + 3z = 1$$

$$2x + 3y - z = 3$$

$$5x + 7y + z = 7$$ 6

(b) Obtain the rank of the matrix :

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & 1 \\ 2 & 0 & -3 & 2 \\ 3 & 3 & -3 & 3 \end{bmatrix}$$ 6

SECTION – II

4. (a) Examine the convergence of the sequence :

$$\left\{\frac{n}{n+1}\right\}$$ 6

(b) Find the n^{th} derivative of $y = \frac{1}{x^2+1}$ 6

(c) If $y = \sin(m\sin^{-1}x)$, prove that

$$(1 - x^2)\, y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)\, y_n = 0$$ 6

5. (a) Sketch the graph of $y = \sqrt{x+3}$ 6

(b) A body with initial temperature of 100°C is allowed to cool in air which remains at a constant temperature of 20°C. It is given that after 10 minutes, the body has cooled to 40°C. Find the temperature of the body after half an hour. 6

(c) Find the Meclaurin's series for $e^{-x/2}$. 6

6. (a) Draw the level curves of the function

$z = -2x - 3y + 5$ at heights k = 1, 2, 5. 6

(b) A point moves along the intersection of the elliptic paraboloid $z = x^2 + 3y^2$ and the plane $y = 1$. At what rate is z changing with x when the point is (2, 1, 7) ? 6

(c) Verify that the function :

$$u(x, t) = e^{c^2\pi^2 t}(Ae^{\pi x} + Be^{-\pi x})$$

is a solution to the heat equation $u_{xx} = \frac{1}{c^2} u_t$, where A, B are constants. 6

SECTION – III

7. (a) If the probabilities are 0.60, 0.25 and 0.18 that a person in a certain income group will invest his money in stock market, in bank deposit or in both respectively. Find the probability that a person in that income group.

(i) Who invests his money in stock market will also invest in bank deposit?

(ii) Who invests in bank deposit will also invest in stock market? 4

(b) If $f(x) = \frac{k}{2^x}$ is a probability distribution for a variable that can take on values x = 0,1,2,3 and 4, find k. 3½

[P. T. O.

8. (a) A random variable X assumes any positive integral value n with a probability proportional to $\frac{1}{2^n}$. Find the expectation of X. 4

(b) What is the probability that an income Tax Officer will find only 2 income tax returns with illegitimate deductions if he randomly selects 6 returns from amongst 18 returns of which 4 contain illegitimate deductions? 3½

9. (a) In 64 randomly selected hours of production, the mean and standard deviation of the numbers of acceptable pieces produced by an automatic stamping machine are $\bar{x} = 962$ and $s = 146$. At 0.05 level of significance does it justify the claim of the machine owner that the average production per hour of the machine is 1000 pieces? 4

(b) In a partially destroyed laboratory record of the correlation analysis of data, the following results are only legible :

Var(x) = 9, regression lines are

$2x - 5y + 16 = 0$ and $6x - 3y = 12$

Find out

(i) The mean values of x and y,

(ii) The correlation coefficient between x and y. 3½