[This question paper contains 6 printed pages.]

Your Roll No. ....

5100 **B**

B.Sc. (Prog.)/ B.Sc. (Hons.) Sem. I

M.A. 107-B – MATHEMATICS

(For Life Sciences)

(Admissions of 2008 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are three Sections.

Attempt any two questions from each Section.

Students are allowed to use calculators.

SECTION – I

1. (a) Show that the cross product of A = {0,1} and B = {0,2,4} is not commutative. 4

   (b) A linear function G = G(t) assumes the value G_1, = 88.3 mg at a time in start t_1 = 14 sec. and the value G_2 = 89.6 mg at t_2 = 39 sec. Find the rate of growth and establish the linear function. 4

   (c) A certain culture of bacteria grows at a rate proportional to the number present. It is found that the number doubles in 4 hours. Find how many bacteria may be expected at the end of 24 hours? 4½

2. (a) Show that for a quadratic polynomial $y = ax^2 + bx + c$, the first difference form a linear function, the second difference remain constant. 4½

(b) How do the following terms behave as $n \to \infty$?

(i) $100\left(\frac{1}{2}\right)^n$ (ii) $\left(\frac{-1}{2}\right)^n$

(iii) $\left(\frac{5}{4}\right)^n$ (iv) $\left(1+10^{-3}\right)^n$ 4

(c) Assume that a population of size 25000 (at a time t =0) grows according to the formula $N=25000 + 45t^2$ where the time t is measured in days. Find the average growth rate in the time intervals.

(i) from t = 0 to t = 2.

(ii) from t = 2 to t = 10. 4

3. (a) Evaluate :

(i) $\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(x-\frac{1}{x}\right)dx$

(ii) $\int_0^1\left(x^2+x+1\right)dx.$ 4

(b) The absolute temperature T of a gas is given by T = CPV, where p is its pressure, v its volume, C is some constant depending on the mass of gas. If $p = (t^2 + 2t)$ and $v = (2t + t^{-1})$ as functions of time t, find the rate of change of T.w. r. t. 't'.4

(c) Find the second derivative of *y w.r.t. x*. 4½

(i) $y = (x^2 + 5x)^2$

(ii) $y = \frac{x}{x-3}$

SECTION – II

4. (a) If :

$$A = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{-1}{2} & \frac{-1}{2} \\ \frac{\sqrt{2}}{2} & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \end{pmatrix}$$

Show that A is an orthogonal matrix. 4

(b) Solve the matrix equation.

$$\begin{pmatrix} x & 3 \\ y & 4 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 43 & -23 \\ -2 & -1 \end{pmatrix}$$

with respect to x and y. 4½

(c) Find the image of points (–2, –7), (–6, 2) under replection in x-axis as well as y-axis. 4

5. (a) If $z = x^y + y^x$, show that :

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$ 4½

(b) Verify that $y = \frac{1}{x}$, $\ln x$ is a solution of

$$\frac{x^2 a^2 y}{ax^2} + \frac{xdy}{ax} - y = \ln x.$$ 4

(c) If :

$$M = \begin{pmatrix} -1 & 5 & 5 \\ 3 & -1 & 5 \\ 3 & 3 & -1 \end{pmatrix}$$ 4

Evaluate M M^T. Where T stands for transpose.

6. (a) Assume that a population groups in such a way that $\frac{1}{N}\frac{dN}{dt}$ remains constant. Let N_1 be the number of individuals at the time t_1. Find N = N (f). 4½

(b) Verify that $z = e^{-c^2\pi^2 t} \sin \pi x$, is a solution of the one-dimensional heat equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial z}{\partial t}$$ 4

(c) If :

$$P = \begin{pmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{pmatrix}$$

be the transitions matrix of a Markov Chain. Find the transition matrix of two-step transitions. 4

SECTION – III

7. (a) The height of plants of a certain species are normally distributed, the mean height being 30cm. and standard deviation being 5 cm. What proportion of plants are greater than 40 cm. in hight.

(Area under the standard normal curve from 0 to 2 = 0.4772). 6

(b) Seven mice are taken and their body weight (x) and length from nose to tail (y) are measured as follows.

Mouse :	1	2	3	4	5	6	7
Weight (x) :	1	4	3	4	8	9	8
Length (y) :	2	5	8	12	14	19	22

Find the Karl Pearson's coefficient of correlation between the two measures. 6½

8. (a) Find the equations of regression lines for the following values of x and y :

x :	1	2	3	4	5
y :	2	5	3	8	7

also estimate y for $x = 10$. 6½

(b) The probability that a person over 60 years old drinks alcohol is 2/5 and the probability that a person over 60 years old has a heart disease is 2/15. The probability that a person over 60 years old has heart disease and drinks alcohol is 1/16. Are the drinking alcohol and 'heart disease' independent events? 6

9. (a) A soap manufacturing company was distribting a particular brand of soap through a large number of retail shops. Before advertisement compaign, the mean sales per week was 140 dozn. After the compaign a sample of 26 shops were taken and the mean sales was found to be 147 dozens with standard deviation 16. can you consider the advertisement effective? 6½

(b) The following are the weights of the 6 subjects in the sample studied by a scientist :

83.9, 99, 63.8, 71.3, 65.3, 79.6

Compute the mean and standard deviation. 6