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Your Roll No................

5192-J

B.Sc. (PHYSICAL SCIENCE)/II Sem. B

Paper—MAPT-101

Mathematics–I (Calculus and Matrices)

(Admission of 2010 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each Section.

Section I

1. (*a*) Show that the set $S = \{(0, 1, 0), (1, 0, 1), (1, 1, 0)\}$ forms a basis for $\mathbf{R}^3$. 6

(*b*) Examine which of the following is a subspace of $\mathbf{R}^2$. Also justify : 6

$$S_1 = \{(a, 0) : a \in \mathbf{R}\}$$

$$S_2 = \{(1, y) : y \in \mathbf{R}\}$$

2. (*a*) Which of the following transformations are linear ? Justify : 6

(*i*) $T(x, y) = 2x - y$

(*ii*) $T(x, y) = x + 1$.

(*b*) Find the image of triangle having vertices (0, 0), (1, 2) and (2, 0) under translation by vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. 6

3. (*a*) Reduce the matrix

$$A = \begin{pmatrix} 5 & 3 & 14 & 4 \\ 5 & 4 & 16 & 5 \\ 1 & -1 & 2 & 0 \end{pmatrix}.$$

to triangular form by elementary row operations and hence determine its rank. 6

(*b*) Find eigen values and eigen vector corresponding to one of them for the matrix 6

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

4. (*a*) Solve the system of equations :

$$2x + 3y + z = 9$$

$$x + 2y + 3z = 6$$

$$3x + y + 2z = 8$$ 6

(*b*) For what value of λ, does the following system of equations have a solution : 6

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2.$$ 6

Section II

5. (*a*) Discuss the convergence of sequence :

$$\left\langle \frac{\sin n}{n} \right\rangle.$$ 6

(*b*) Find the *n*th derivative of :

$$y = \frac{x}{2x^2 + 3x + 1}.$$ 6

(*c*) If

$$y = \sin (m \sin^{-1} x),$$

prove that :

$$(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 - m^2)y_n = 0.$$ 6

6. (*a*) Sketch the graph of function

$$f(x) = \sin 2x, \quad x \in \mathbf{R}.$$ 6

(*b*) Find the Maclaurin series expansion of $y = x \cos x$, assuming that $\lim_{n \to \infty} R_n(x) = 0$. 6

(*c*) In a reserve forest, there is capacity to maintain 500 elephants. Initially there were 20 elephants. Within 2 years time, the number rose to 30. Assuming logistic growth model, find out how much time it will take for their population to grow to 200. 6

7. (*a*) Draw the level curves of height $k = 0, 1, 2$ for the surface $z = \sqrt{4 - x^2 - y^2}$ 6

(*b*) If

$$u = \frac{1}{\sqrt{x^2 + y^2 + z^2}},$$

show that :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$ 6

(*c*) Verify that the function $\omega = e^{-c^2k^2t}(a \cos kx + b \sin kx)$ is a solution of diffusion equation. 6

8. (*a*) For what value of x, can we approximate e^x by $1 + x + \frac{x^2}{2}$. Correct to four decimal places ? 6

(*b*) Show that the sequence $<a_n>$ defined by

$$a_1 = 1 \qquad a_{n+1} = \sqrt{2 + a_n} \quad \forall_n \geq 1$$

converges. Also find its limit. 6

(c) Show that the sequence $\left\langle \frac{\lfloor n}{n^n} \right\rangle$ converges. 6

Section III

9. (a) Prove that for any two complex numbers z_1 and z_2

$$| |z_1| - |z_2| | \leq |z_1 - z_2|.$$ 4

(b) Form an equation of the lowest degree with rational coefficients having $2+\sqrt{3}$ and $\sqrt{5}-2$ as two of its roots. 3½

10. (a) Find the centre and radius of circle whose equation is

$$|z - i| = 3\,|z + i|.$$ 3½

(b) Let z_1, z_2, z_3 be the affixes of the points P, Q, R respectively. If $|z_1| = |z_2| = |z_3|$ and $z_1 + z_2 + z_3 = 0$, prove that the triangle PQR is equilateral. 4

11. (*a*) Show that

$$(1 + \cos\theta + i\sin\theta)^n + (1 + \cos\theta - i\sin\theta)^n$$

$$= 2^{n+1} \cos^n \frac{\theta}{2} \cos^n \frac{\theta}{2}. \qquad 3\tfrac{1}{2}$$

(*b*) Solve the equation :

$$z^4 + z^3 + z^2 + z + 1 = 0. \qquad 4$$

12. (*a*) Using elementary row operations find the inverse of matrix : 4

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{pmatrix}.$$

(*b*) Express the vector $\begin{pmatrix} 8 \\ 7 \end{pmatrix}$ as a linear combination of vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. 3½