[This question paper contains 4 printed pages.]

Your Roll No. ....

5184-L **B**

B.Sc. (Prog.) Structure Courses Mathematical Sciences/II Sem.

Paper : STP-202 : Statistical Method–I

(Admissions of 2011 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *Six* questions.

All questions carry equal marks.

1. (*a*) The distribution function of a random variable X is given by : 6½

$$F(x) = \begin{cases} 1-(1+x)e^{x}, & x \geq 0 \\ 0 & , x < 0 \end{cases}$$

Find (*i*) p.d.f of X,

(*ii*) $P(x \geq 2)$, (iii) $P(-2 < x \leq 3)$.

(*b*) A random variable X has the following probability function :

x	0	1	2	3
$p(x)$	0.1	0.3	0.5	0.1

If $Y = X^2 + 2X$, find

(*i*) the probability function of Y

(*ii*) mean and variance of Y. 6

2. (*a*) Let the p.d.f. of a random variable X be :

$$f(x) = \begin{cases} \frac{1}{6}, -3 \le x \le 3, \\ 0, \quad \text{otherwise.} \end{cases}$$

Find the p.d.f. of (i) $Y = X^2$, (ii) $Y = 3X + 1$. 6

(*b*) The joint probability density function of the two dimensional random variable (X, Y) is given by :

$f(x, y) = x^3y^3/16,\ 0 \le x \le 2,\ 0 \le y \le 2.$

Find (*i*) marginal p.d.f. of X.

(*ii*) conditional p.d.f. of Y given X.

(*iii*) $P(X = 1;\ Y = 1)$. 6½

3. (*a*) State and prove addition theorem of expectation for two random variables. Hence, find mean of $Y = 2X_1 - 3X_2$ where X_1, X_2 are random variables with means 3 and 5 respectively. 6

(*b*) Find the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability *p* of success in each trial. 6½

4. (*a*) Define the characteristic function of a random variable. Show that the characteristic function of the sum of two independent random variables is equal to the product of their characteristic functions. 6

(*b*) Let the random X assume the value r with probability law :

$P(X = r) = pq^{r-1}, r = 1, 2, 3, ...$

Find the m.g.f. of X and hence its mean and variance. 6½

5. (*a*) Obtain the mode of Poisson distribution with mean λ. Hence find it if

(*i*) $\lambda = 3.5$, (*ii*) $\lambda = 5$. 6½

(*b*) Define negative binomial distribution. Obtain Poisson distribution as a limiting case of the negative binomial distribution. 6

6. (*a*) Define 'Lack of memory' property. Which discrete distribution satisfies this property ? Give the proof also. 6

(*b*) Let $X \sim B(n, p)$. Show that

$$\mu_{r+1} = pq\left(n_r \mu_{r-1} + \frac{d\mu_i}{dp}\right)$$

Where μ_r is r^{th} moment about mean. Hence find variance. 6½

7. (*a*) Let $X_i \sim \text{gamma}(\lambda_i, a)$, $i = 1, 2, ..., n$ be independent random variables. Obtain the distribution of $X_1 + X_2 + ... + X_n$ and identify it. 6

(*b*) For a normal distribution with mean μ and variance σ^2, show that the mean deviation from the mean 'μ' is equal to $\sigma\sqrt{2/\pi}$. What will be the mean deviation from median ? 6½

8. (*a*) A random variable X takes the values -1, 1, 3, 5 with associated probabilities 1/6, 1/6, 1/6 and 1/2. Find by direct computation $P(|X-3| \geq 1)$ and also find an upper bound to this probability by applying Chebychev's inequality. 6

(*b*) State Central limit theorem for iid variables.

Let $X_1, X_2, \ldots X_n$ be iid Poisson variates with parameter λ. Use central limit theorem to find asymptotic distribution of $S_n = X_1 + X_2 + \ldots + X_n$. 6½