

[This question paper contains 6 printed pages.]

1905

Your Roll No.

B.Sc. (Prog.)

E

M.A. 107-B - MATHEMATICS

(For Life Sciences)

(Admissions of 2008 & onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*There are three sections in this question paper.
Attempt any two questions from each Section.
Students are allowed to use simple calculator.
Please indicate the section you are attempting at
the appropriate place and do not intermix the
sections. The questions should be numbered in
accordance to the number in the question paper.*

SECTION - I

1. (a) Consider a spherical cell of volume V and surface S . Express V as a function of S . What type of function is it? How does doubling S influence V ? (4)

P.T.O.

- (b) 6000 Indians were tested for A, B, O, blood groups, 2527 had the antigen A, 2234 the antigen B, and 1846 no antigen. How many individual had both antigen. (4½)

(c) Expand $\frac{e^x + e^{6x}}{e^{4x}}$ in ascending powers of x . (4)

2. (a) Evaluate (i) $\lim_{x \rightarrow \infty} \frac{x^2 - 2x + 3}{5 - 2x^2}$ and (ii) $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^x$ (4)

(b) Find $\frac{dy}{dx}$, if x and y are related by the equation
 $x^2 \sin y = y^2 \sin x$. (4½)

- (c) If the distance S covered by a particle in t seconds is given by $S = \sqrt{t}$, show that the acceleration is negative and is proportional to the cube of the velocity. (4)

3. (a) Integrate the following :

(i) $\int \frac{\tan 2x}{\sec 2x} dx$

(ii) $\int \frac{2x+1}{\sqrt{x^2+x+1}} dx$ (4)

(b) Evaluate the following :

$$(i) \int_0^{\pi/4} (2 \sec^2 x + x^3 + 2) dx$$

$$(ii) \int_0^1 \frac{1}{\sqrt{x+1} + \sqrt{x}} dx \quad (4\frac{1}{2})$$

(c) If $f'(x) = x - \frac{1}{x^2}$ and $f(1) = \frac{1}{2}$, find $f(x)$. (4)

SECTION - II

4. (a) Express the matrix equation

$$x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} -1 \\ 6 \end{bmatrix}, \text{ as a system of equations and} \\ \text{hence solve it.} \quad (4)$$

(b) Find the image of a point (3, -2) under the following transformation using matrix multiplication,

(i) dilation by a scale factor 5.

(ii) rotation through an angle 30° in the counter clockwise direction. (4½)

(c) Find x and y so that the matrix, $A = \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$

may satisfy the condition $AA^T = I = A^T A$. (4)

5. (a) If $u = x^y \cdot y^x$, prove that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = (x + y + \log u)u \quad (4)$$

- (b) Verify that $y = \frac{4 \ln x}{x^2}$, ($x > 0$) is a solution of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 4y = 0 \quad (4\frac{1}{2})$$

- (c) Show that $w = \ln(2x + 2ct)$ is a solution of the

wave equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$, where c is the

velocity with which the wave propagates. (4)

6. (a) For $z = \frac{x^2}{2y} + \frac{y^2}{2x}$, verify that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$. (4)

- (b) A certain culture of bacteria grows at a rate proportional to the number present. It is found that the number doubles in 4 hours. How many bacteria may be expected at the end of 24 hours. (4½)
- (c) The volume V of a right circular cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height. Suppose that r has a constant value 8, but h varies. Find the rate of change of V with respect to h at the point where $h = 10$. (4)

SECTION - III

7. (a) An incomplete frequency distribution is given below :

<i>Variable</i>	0-10	10-20	20-30	30-40	40-50	50-60
<i>Frequency</i>	10	?	25	30	?	10

Given that the total frequency is 100 and median is 32, find the missing frequencies. (6½)

- (b) The heights of plants of a certain species are normally distributed, the mean height being 30 cm and the standard deviation being 5 cm. What proportion of plants are (i) greater than 40 cm in height and (ii) less than 40 cm in height ?
- (Area under the standard normal curve from 0 to 2 is 0.4772) (6)

8. (a) The weights of a calf taken at weekly intervals are given below :

Age (x) (in weeks)	1	2	3	4	5
Wt. (y) (in kg.)	30	34	37	43	45

Fit a straight line to the above data and then give an estimate of weight for the calf when it becomes 7 weeks old. (6½)

- (b) A bag contains 3 red, and 5 white balls. Two balls are drawn one by one without replacement. What is the probability of drawing red balls in both trials? (6)
9. (a) A sample of 50 associate degree graduates (sample A) and a sample of 60 baccalaureate graduates (sample B) yielded the following means and standard deviations. (6½)

Sample	Mean	Standard Deviation
A	52.5	10.5
B	49.6	11.2

On the basis of these data, what should the researcher conclude at 5% level of significance.

- (b) For the probability distribution given below, find (i) k, (ii) the expected value $E(X)$ and (iii) the variance σ_x^2 .

x	1	2	3	4	5	6
p(x)	k	2k	3k	4k	5k	6k

(6)

(100)