

[This question paper contains 6 printed pages.]

1904

Your Roll No.

B.Sc. Prog. / I

E

MA-107 (a) – MATHEMATICS – I

(For Physical Sciences)

(NC – Admissions of 2008 & onwards)

Time : 3 Hours

Maximum Marks : 75

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

*There are **three** sections in this question paper.
Attempt any **two** questions from each Section.*

SECTION I

1. (a) A ship is being pushed by a tugboat with a force of 300 pounds along the negative y-axis while another tugboat is pushing the negative x-axis with a force 400 pounds. Find the magnitude and sketch the direction of the resultant force. (4)

P.T.O.

(b) Prove that $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -x \\ y \end{bmatrix}$

is a linear transformation. Find images of the points $(2,3)$ and $(-5,-4)$ under this transformation. (8)

2. (a) Which of the following are subspaces of \mathbb{R}^2 :

(i) $V_1 = \{(x, 0): x \text{ is a real number}\}$

(ii) $V_2 = \{(1, y): y \text{ is a real number}\}$

(iii) $V_3 = \{(a, 2a): a \text{ is a real number}\}$ (6)

(b) Solve the system of equations

$$x + y + z = 7$$

$$x + 2y + 3z = 16$$

$$x + 3y + 4z = 22 \quad (6)$$

3. (a) Find inverse of a matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$$

using elementary row operations. (6)

(b) Find the rank of a matrix :

$$\begin{bmatrix} 2 & 4 & 5 \\ 1 & -1 & 2 \\ 6 & 12 & 15 \end{bmatrix} \quad (6)$$

SECTION II

4. (a) Discusses convergence of the sequence $\left\{ \frac{\sin n}{n} \right\}$.
(6)

(b) Write 5 terms of the sequence given by

$$a_1 = 1, a_{n+1} = \sqrt{2a_n} \quad (5)$$

(c) If $y = \tan^{-1}x$, prove that $(1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = 0$.

Hence show that

$$(1+x^2)y_{n+2} + 2(n+1)xy_{n+1} + n(n+1)y_n = 0 \quad (7)$$

5. (a) An aluminum beam was brought from the outside cold into the machine shop where the temperature was held at 65°C . After 10 minutes the beam was warmed to 35°C and after another 10 minutes it was 50°C . Use Newton's law of cooling to estimate the beams initial temperature. (6)

(b) Find Maclaurin's series expansion of $y = \cos x$

by assuming $\lim_{n \rightarrow \infty} R_n = 0$ (6)

(c) Draw the graph of (i) $y = \frac{1}{2}x^2$ (ii) $y = e^x + 2$.

(6)

6. (a) Show that :

$$z = e^x \sin y + e^y \cos x$$

is the solution of Laplace's equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$

(6)

(b) Draw the level curves of height k , where $k = 0$, $k = 2$, and $k = 4$ for the surface:

$$Z = 1 - x - 2y$$
 (6)

(c) Find n th derivative of $y = \frac{1}{ax + b}$. (6)

SECTION III

7. (a) A person repairing an air-conditioner claims that the probability that the motor is alright is 0.82, that the fan is alright is 0.64 and that they both are alright is 0.41. Do you think that his claim is justified? (3½)

(b) Find mean and variance of Poisson distribution.

(4)

8. (a) For the following probability distribution, find (i) k
(ii) the expected value $E(x)$

X:	1	2	3	4	5	6
P(x)	k	$2k$	$3k$	$4k$	$5k$	$5k$

(4)

(b) Fit a straight line to the following data taking y as the dependent variable:

X:	6	8	8	4	9
Y:	5	6	7	5	8

(3½)

9. (a) Calculate coefficient of correlation between the weights in pounds of brother (x) and sister (y) for the following data:

X:	20	30	25	25	35	40
Y:	15	25	15	20	20	30

(4)

- (b) A machine makes engine parts with axle diameter 0.7 inches. A random sample of 10 parts shows a mean diameter of 0.742 inches with a standard deviation of 0.04. Test whether his work is meeting the specifications. (3½)