[This question paper contains 6 printed pages.]

1962 *Your Roll No.* ....

B.Sc. / I **E**

MATHO-PHYSICS

MP-201 : Mathematics – I

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all selecting* **two** *questions from Section* **I, three** *questions from Section* **II,** *and* **one** *question from Section* **III.**

SECTION I

Attempt any **two** *question from this section.*

1\. (a) Define a parabola and find its equation in standard form. (7)

(b) Sketch the ellipse : $9x^2 + y^2 = 9$,

and label the foci, the vertices and the ends of the minor axis. (6)

(c) Find the equation for a hyperbola that satisfies the conditions Vertices $(\pm 1, 0)$, asymptotes $y = \pm 2x$. (6)

P.T.O.

2. (a) Prove that curl grad $\phi = 0$ for any scalar function $\phi(x, y, z)$. (7)

(b) Prove that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. (6)

(c) Find the angle between the vectors

$\vec{u} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{v} = -3\hat{i} + 6\hat{j} - 6\hat{k}$ (6)

3. (a) Prove that div curl $\vec{A} = 0$ (7)

(b) If $\phi(x, y, z) = 3x^2y - y^2z^2$, find $\nabla \phi$ at the point $(1, -2, -1)$. (6)

(c) Evaluate (i) $\text{div}\left(\vec{a} \times \vec{r}\right)$

(ii) $\text{curl}\left(\vec{a} \times \vec{r}\right)$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{a}$ is a constant vector (6)

SECTION II

Attempt all **three** *questions in this section.*

4. (a) Find all the asymptotes of the curve

$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$ (8)

(b) Trace the curve

$$x(x^2 + y^2) = a(x^2 - y^2), \qquad (a > 0) \qquad (10)$$

Or

(a) Find the position and nature of the double points on the curve.

$$2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0 \qquad (8)$$

(b) Trace the curve

$$x(x^2 + y^2) - ay^2 = 0, \; a > 0 \qquad (10)$$

5. (a) Evaluate

$$\int \frac{dx}{(1+x)\sqrt{x^2 - 1}} \qquad (8)$$

(b) Find the surface area of the solid obtained by revolving arc of the parabola.

$$y^2 = 4ax, \qquad 0 \le x \le a$$

about x - axis. (8)

Or

(a) Obtain reduction formula for

$\int (a^2 + x^2)^{\frac{n}{2}} dx$, where n is any positive integer.

P.T.O.

Hence or otherwise, evaluate $\int \left(a^2 + x^2\right)^{\frac{5}{2}} dx$. (8)

(b) Find the volume of the solid obtained by revolving the arc of the curve.

$ay^2 = x(x + a)^2, \quad 0 \le x \le a.$

about x - axis. (8)

6. (a) State and prove Lagrange's mean value Theorem. (10)

(b) Verify Rolle's Theorem for the function

$f(x) = (x-1)(x-2)(x-3)$ in $[1,3]$. (6)

(c) Examine the continuity of the function.

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$ (8)

Or

(a) Separate the intervals in which the function f(x) defined by

$$f(x) = 2x^3 - 9x^2 + 12x + 1$$

is increasing or decreasing. (8)

(b) Show that function f(x) defined by

$$f(x) = \sin\frac{1}{x}, \qquad x \in]\,0, 1]$$

is not uniformly continuous in $0 < x \le 1$. (8)

(c) State Rolle's Theorem.

Show that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two district roots in [0, 1]. (8)

SECTION – III

Attempt any **one** *question from this section.*

7. (a) Show that a necessary condition that the points A, B, C representing the numbers z_1, z_2, z_3 respectively on the Argand Plane to be the vertices of an equilateral triangle is that

$$\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0 \qquad (6)$$

(b) Solve the equation

$$1 + z + z^2 + z^3 + z^4 + z^5 = 0 \qquad (5)$$

(c) Prove that

$$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta \qquad (5)$$

8. (a) Solve the equation

$$3x^3 + 11x^2 + 12x + 4 = 0,$$

being given that the roots are in H.P. (5)

(b) Find the equation whose roots are the cubes of the roots of the equation

$$x^4 - x^3 + 2x^2 + 3x + 1 = 0$$ (5)

(c) If α, β, γ be the roots of the equation :

$x^3 - px^2 + qx - r = 0$, find the value of

(i) $\sum \alpha^2 \beta$ (ii) $\sum \frac{\alpha}{\beta}$ (6)