[This question paper contains 4 printed pages.]

4618 *Your Roll No.* ....

B.Sc. Prog./II **AS**

MA 201 – MATHEMATICS – I

(Calculus and Geometry)

(For Physical Sciences/Applied Sciences)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *sections are compulsory.*

Attempt any **two** *parts in each question of Sections* **I** *and* **II**. *In Section* **III** *attempt* **four** *parts in Q.* **5**, *part (a) being compulsory.*

SECTION I

1. (a) (i) Give the geometrical representation of subtraction of two complex numbers. (5½)

(ii) Find the equation of the straight line joining the points A and B which have $2 + 3i$, $-3 + i$ as affixes. (7)

(b) (i) If the sum of two roots of the equation

$$4x^4 - 24x^3 + 31x^2 + 6x - 8 = 0$$

is zero, find all the roots of the equation. (7)

P.T.O.

(ii) If α, β, γ, be the roots of the equation

$$x^3 + qx + p = 0$$

then prove that

$$\frac{\alpha^5+\beta^5+\gamma^5}{5} = \frac{\alpha^3+\beta^3+\gamma^3}{3} \times \frac{\alpha^2+\beta^2+\gamma^2}{2}$$

(5½)

(c) (i) Using DeMoivre's theorem, solve the equation $z^9 + z^5 + z^4 + 1 = 0$. (7)

(ii) If n is a positive integer, prove that :

$$\left(1+i\sqrt{3}\right)^n + \left(1-i\sqrt{3}\right)^n = 2^{n+1}\cos\frac{n\pi}{3}$$ (5½)

SECTION II

2. (a) (i) Using $\in$–δ approach, show that

$$\lim_{x\to 3}\left(2x+1\right) = 7.$$ (5)

(ii) Show that f is continuous but not derivable at the origin, where

$$f(x) = x\frac{e^{1/x}-1}{e^{1/x}+1},\ x \neq 0$$
$$= 0 \qquad ,\ x = 0$$ (5)

(b) (i) State Rolle's theorem. Show that there is no real number k for which the equation $x^2 - 2x + k = 0$ has two distinct roots in [0, 1]. (5)

(ii) Prove that uniform continuity implies continuity. (5)

(c) (i) State Darboux Theorem. Explain it with the help of an example. (5)

(ii) Find out the points of inflexion of the curve $y = x^4 - 4x^3 - 18x^2 + 1$. (5)

3. (a) Find the asymptotes of the following curve

$$(y - a)^2(x^2 - a^2) = x^4 + a^4. \quad (10)$$

(b) (i) Find the position and nature of the double points of the following curve

$$x^3 + y^3 = 3axy. \quad (5)$$

(ii) Trace the following curve

$$\gamma = a(1 - \cos\theta). \quad (5)$$

(c) Trace the following curve

$$x^2(x^2 + y^2) = a^2(x^2 - y^2). \quad (10)$$

4. (a) (i) Obtain the reduction formula for $\int \tan^n x dx$. (5)

(ii) If $I_n = \int_0^{\pi/4} \tan^n \theta d\theta$, then show that

$$I_n + I_{n-2} = \frac{1}{n-1}.$$

Hence deduce the value of I_5. (5)

(b) A curve is gien by the equations

$$x = a(\cos + \theta\sin\theta), \; y = a(\sin\theta - \theta\cos\theta).$$

Find the lenth of the arc from $\theta = 0$ to $\theta = \alpha$. (10)

P.T.O.

(c) Find the volume of the solid generated by resolving the ellipse $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ about the Y-axis. (10)

SECTION III

Attempt **four** *parts of this question*
Part (a) is compulsory.

5. (a) Rotate the axes of co-ordinate to get rid of the xy-term from the equation

$$x^2 - xy + y^2 - 2 = 0$$

and trace the conic. (12)

OR

Rotate the axes of co-ordinate to get rid of the xy-term from the equation

$$x^2 + 4xy - 2y^2 - 6 = 0$$

and trace the conic. (12)

(b) Find an equation of the parabola with vertex (1,1) and directrix $y=-2$. (5)

(c) If $\vec{A}=5t^2\hat{i}+t\hat{j}-t^3\hat{k}$ and

$\vec{B}=\sin t\,\hat{i}-\cos t\,\hat{j}$,

find $\frac{d}{dt}\left(\vec{A}\cdot\vec{B}\right)$ and $\frac{d}{dt}\left(\vec{A}\times\vec{B}\right)$. (5)

(d) If $\phi = 2x^3y^2z^4$, find $\nabla\cdot\nabla\phi$ (or div gradϕ). (5)

(e) If $\vec{A}=x^2y\hat{i}-2xz\hat{j}+2yz\hat{k}$, find curl curl$\vec{A}$. (5)

(300)****