[This question paper contains 4 printed pages.]

4619 *Your Roll No.* ....

B.Sc. Prog./II **AS**

MA-202 – MATHEMATICS (Algebra & Differential Equations)

(For Physical Sciences/Applied Physical Sciences)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from ach question.*
All *questions are compulsory.*

UNIT I

1. (a) Let $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} a, b, c, d \in \mathbb{R},\ ad-bc \in \mathbb{R}^*, \text{ the set of non zero rational numbers} \right\}$

Show that G is a group under matrix multiplication. Is G abelian ? Justify. (7½)

(b) Let $A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ and G be the group as in (a). Show that $A \in G$ and find the order of A. (7½)

(c) Let H, K be subgroups of a group G. Is $H \cup K$ a subgroup of G ? Justify. (7½)

2. (a) Let G be a group of order 98 and $a \in G$ such that $0(a) = 49$. Now let $H = \langle a \rangle$. Show that H is a normal subgroup of G. (7½)

(b) Let G be a cyclic group of order n. If 'a' $\in$ G is a generator of G and K is an integer co-prime to n, then show that a^K also generates G. (7½)

(c) Let $G = \mathbb{Z}$, the group of integers. Let $H = 3\mathbb{Z}$, Is G/H defined ? Write explicitely the elements of G/H if your answer is yes. Also find order of G/H. (7½)

3. (a) State Fermat's theorem. Use it to find the remainder when 2^{50} is divided by 7. (7½)

(b) (i) Express $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ as a product of transpositions/2-cycles.

(ii) Compute $\alpha^{-1}\beta\alpha$ when

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 5 & 4 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$$

(7½)

(c) Show that the subgroups of cyclic groups are cyclic. (7½)

UNIT II

4. (a) Solve :

(i) $(4xy^2 + 6y)dx + (5x^2y + 8x)dy = 0$,

(ii) $p^2 + 2py \cot x = y^2$, $p = \frac{dy}{dx}$. (6,5½)

(b) Solve by the method of variation of parameters :

$$\frac{d^2y}{dx^2} + 4y = \sec^2 2x .$$ (11½)

(c) Prove that there exists two linearly independent solutions $y_1(x)$, and $y_2(x)$ of the second order homogeneous linear differential equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0 ,$$

where $a_0(x) \neq 0$ for any x in (a, b) and a_0, a_1, a_2 are continuous real-valued functions of x defined on (a,b),

such that every solution y(x) may be written as

$$y(x) = c_1y_1(x) + c_2y_2(x), \; x \in (a,b),$$

where c_1 and c_2 are suitably chosen constants.

(11½)

5. (a) Solve :

$$x^3\frac{d^3y}{dx^3} + 2x\frac{dy}{dx} - 2y = x^2 \log x + 3x$$ (11)

(b) Solve :

$$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz = 0.$$ (11)

P.T.O.

(c) A 6 lb. weight is attached to the lower end of a coil spring suspended from the ceiling, the spring constant of the spring being 27 lb/ft. The weight comes to rest in its equilibrium position, and beginning at $t = 0$ and external force given by $F(t) = 12 \cos 20t$ is applied to the system. Determine the resulting displacement as a function of the time, assuming damping is negligible. (11)

UNIT III

6. (a) Find the general integral of

$$px(z - 2y^2) = (z - qy)(z - y^2 - 2x^2),$$

where $p = \frac{\partial z}{\partial z}$, $q = \frac{\partial z}{\partial y}$. (11)

(b) Find the complete integral of

$$p^2q(x^2 + y^2) = p^2 + q$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. (11)

(c) Show that the equations $f(x, y, p, q) = 0$ and $g(x, y, p, q) = 0$ are compatible if

$$\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0.$$

Also show that the equaions $p = P(x, y)$, $q = Q(x, y)$ are compatible if

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

(11)