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Your Roll No................

5189

B.Sc. (PHYSICAL SCIENCE)/III Sem. B

Paper—MAPT-303

(Algebra)

(Admission of 2010 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

Unit I

1. (*a*) Define a group and prove that the set

$$G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in R, a \neq 0 \right\}$$

is a group under matrix multiplication. Is it an infinite abelian group ? 6

(*b*) Let G be a group. Prove that $Z(G) = \bigcap_{a \in G} C(a)$ where $Z(G)$ is the centre of the group G and $C(a)$ is the centralizer of '*a*' in G. 6

P.T.O.

(c) Consider the element $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ in SL(2, R). What is the order of A ? If we view $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ as a member of SL(2, Z_p) (p-prime), what is the order of A ? 6

2. (a) Prove that Z_{30} is a cyclic group. Write down all the subgroups of Z_{30} indicating their orders. 6

(b) Prove that the group (Q, +) of rational numbers under addition is not cyclic. 6

(c) (i) Define an odd permutation and an even permutation and determine whether the permutation (1256743) is odd or even.

(ii) Let α and β belong to S_n. Prove that $\alpha^{-1}\beta^{-1}\alpha\beta$ is an even permutation. 6

3. (*a*) State Lagrange's Theorem for finite groups. What about the converse ? Justify your answer by giving some example. 6

(*b*) Suppose G is a group with order $|G| = pq$, where p and q are prime. Prove that every proper subgroup of G is cyclic. 6

(*c*) Define a normal subgroup of a group G and prove that SL(2, R) is a normal subgroup of GL(2, R). 6

Unit II

4. (*a*) Define an ideal of a ring R and prove that intersection of two ideals of a ring is an ideal but union is not so. 6½

(*b*) Prove that every field is an integral domain. 6½

(*c*) Define a unit and zero divisors in a ring R and give an example of a non-zero element of a ring which is neither a unit nor a zero-divisor. 6½

Unit III

5. (*a*) Prove that the set $\{a_2x^2 + a_1x + a_0 | a_0, a_1, a_2 \in R\}$ is a subspace of the space of all polynomials with real coefficients over R. Prove that $\{1, x, x^2\}$ is a basis for this subspace. 6½

(*b*) Define a basis of a vector space over a field F and prove that every element of a vector space is uniquely expressible as a linear combination of elements of the basis. 6½

(*c*) Define the linear span of a subset S of a vector space V(F) and prove that the linear span of S is a subspace of V(F) containing the set S. 6½

6. (*a*) Let $T : R^3 \rightarrow R^3$ be a linear operator defined by

$$T(x, y, z) = (x + 2y + z, y + z, -x + 3y + 4z).$$

Find out the matrix of T relative to the standard ordered basis $\beta = \{e_1, e_2, e_3\}$ of R^3. 6½ (*b*) If $T : R^2 \to R^3$ be a linear transformation defined by $T(x, y) = (x, x + y, y)$, then find the range, rank, kernel and nullity of T. 6½

(*c*) (*i*) Define a linear tansformation of a vector space V(F) to a vector space U(F).

(*ii*) Which of the following maps T from $R^2 \to R^2$ are linear transformations ? 6½

(I) $T(x_1, x_2) = (1 + x_1, x_2)$

(II) $T(x_1, x_2) = (x_2 - x_1, 0)$.