This question paper contains 4+1 printed pages]

Your Roll No. ....

5122

B.Sc. Prog./II **B**

MA–202–MATHEMATICS–II–Algebra

& Differential Equations

(For Physical Sciences/Applied Physical Sciences)

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks* : 112

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

All questions are compulsory.

UNIT I

1. (*a*) Let $G = \left\{ \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \middle| n \in \mathbf{Z} \right\}$. Show that G is a group under matrix multiplication. 7½

(*b*) Let G be the group as in (*a*) and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Compute A^2, A^{-2}, A^3, A^{-3}. Using the information provided by this computation, show that G is a cyclic group generated by $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. 7½

P.T.O.

(c) Let H be a subgroup of a group G and $a, b \in G$. Show that $Ha \cap Hb = \phi$ or $Ha = Hb$. 7½

2. (a) Let G be a group and $a, b \in G$. Let $ab = ba$, $o(a) = 7$ and $o(b) = 11$, Find $o(ab)$. 7½

(b) Let Z(G) denote the center of a group G and H be a subgroup of G such that $H \subseteq Z(G)$. Show that $Hg = gH$ for all $g \in G$. 7½

(c) Let $G = \{\pm 1, \pm i, \pm j, \pm k\}$, the group of quaternions. Let $H = \{\pm 1, \pm i\}$. Show that H is a normal subgroup of G and G/H is cyclic. 7½

3. (a) Let $\alpha = (12)(34)$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \end{pmatrix}$. Compute $\alpha^{-1}\beta\alpha$. Also show that α and β do not commute. 7½

(b) Show that a subgroup of a cyclic group is cyclic. 7½

(c) Let H, K be two subgroups of a group G such that $o(H) = 12$ and $o(K) = 35$. Find the order of $H \cap K$. 7½

UNIT II

4. (a) Solve :

(i) $[y^2(x+1)+y]dx+(2xy+1)dy=0$,

(ii) $y=2px+f(xp^2)$, $P=\dfrac{dy}{dx}$. 6,5½

(b) Given that $y=x+1$ is a solution of :

$$(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{dy}{dx}+3y=0,$$

find a linearly independent solution by reducing the order. Write the general solution. 11½

(c) Show that two solutions $y_1(x)$ and $y_2(x)$ of the second order homogeneous linear differential equation :

$$a_0(x)\frac{d^2y}{dx^2}+a_1(x)\frac{dy}{dx}+a_2(x)\ y=0,$$

where a_0, a_1, a_2 are continuous real-valued functions of x defined on (a, b) and $a_0(x)\neq 0$ for any x in (a, b), are linearly dependent if and only if their Wronskian is identically zero. 11½

5\. (a) Solve :

$$2\frac{dx}{dt} + 4\frac{dy}{dt} + x - y = 3e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} + 2x + 2y = e^t$$ 11

(b) Solve :

$$yz(y + z)dx + xz(x + z)dy + xy(x + y)dz = 0.$$ 11

(c) A 16-lb weight is placed upon the lower end of a coil spring suspended from the ceiling and comes to rest in its equilibrium position, thereby stretching the spring 8 in. At time $t = 0$ the weight is then struck so as to set it into motion with an initial velocity of 2ft./sec, directed downward. The medium offers a resistance in pounds numerically equal to $6\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. Determine the resulting displacement of the weight as a function of time. 11

UNIT III

6. (*a*) Find the general integral of :

$$(y + zx)\, p - (x + yz) q = x^2 - y^2.$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$ 11

(*b*) Find the complete integral of :

$$p^2 q^2 + x^2 y^2 = x^2 q^2 (x^2 + y^2),$$

where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 11

(*c*) Show that the equations :

$$xp = yq,\ z(xp + yq) = 2xy$$

are compatible and solve them. 11