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Your Roll No. ....

5121

B.Sc. Prog./II B

MA 201 – MATHEMATICS – I

(Calculus and Geometry)

(For Physical Sciences/Applied Sciences)

(Admissions of 2005 and onwards)

Time : 3 *Hours* *Maximum Marks* : 112

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

Attempt any *two* parts in each question of sections **I** and **II**. In section **III** attempt *four* parts in Q. **5**, part **(*a*)** being compulsory.

Section I

1. (*a*) (*i*) Find the equation of the circle described on the join of the points with affixes $1 + 2i$ and $5 - 6i$ as the extremities of a diameter. Also find its radius and affix of its centre. 7

(ii) Give the geometrical representation of multiplication of two complex numbers. 5½

(b) (i) Solve the equation :

$$3x^4 - 40x^3 + 130x^2 - 120x + 27 = 0$$

given that the product of two of its roots is equal to the product of the other two. 7

(ii) If α, β, γ be the roots of the equation :

$$x^3 + 2x^2 + 1 = 0,$$

find the value of $\alpha^{-4} + \beta^{-4} + \gamma^{-4}$. 5½

(c) (i) Using De Moivre's theorem, solve the equation :

$$z^4 + z^3 + z^2 + z + 1 = 0.$$ 7

(ii) If

$$(1 + z)^n = c_0 + c_1 z + \dots\dots\dots + c_n z^n,$$

where n is a positive integer, show that :

$$c_0 - c_2 + c_4 - \dots\dots = 2^{n/2} \cos\left(\frac{n\pi}{4}\right),$$

$$c_1 - c_3 + c_5 - \dots\dots = 2^{n/2} \sin\left(\frac{n\pi}{4}\right).$$ 5½

Section II

2. (*a*) (*i*) Using $\in - \delta$ approach, show that :

$$\lim_{x \to 1} (3x - 2) = 1.$$ 5

(*ii*) Show that f is continuous but not derivable at the origin, where :

$$f(x) = x\frac{e^{1/x} - 1}{e^{1/x} + 1} \quad , \quad x \neq 0$$

$$= 0 \quad , \quad x = 0$$ 5

(*b*) (*i*) State Darboux Theorem. Explain it with the help of an example. 5

(*ii*) Show, with the help of an example, that continuity need not imply uniform continuity. 5

(*c*) (*i*) Provide the geometrical interpretation of Lagrange's Mean-Value Theorem. 5

(*ii*) Choose 'a' such that the function :

$$f(x) = x^3 + ax^2 + 1$$

has an inflexion point at $x = 1$. 5

3. (*a*) Find the asymptotes of the following curve :

$$x^2y - xy^2 + xy + y^2 + x - y = 0.$$ 10

(*b*) (*i*) Determine the position and nature of the double points of the curve :

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0.$$ 5

(*ii*) Trace the following curve :

$$r = a \sin 2\theta.$$ 5

(*c*) Trace the following curve :

$$y^2(a + x) = x^2(3a - x).$$ 10

4. (*a*) Obtain the reduction formula for :

$$\int \sin^m x \cos^n x \, dx,$$

m, n being positive integers. 10

(*b*) Show that the length of the loop of the curve $3ay^2 = x(x - a)^2$ is $\frac{4a}{\sqrt{3}}$. 10

(c) Find the volume of the solid generated by revolving the ellipse :

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

about the Y-axis. 10

Section III

5. Attempt *four* parts of this question, Part (*a*) is compulsory :

(*a*) Rotate the axes of co-ordinate to get rid of the xy-term from the equation :

$$x^2 - xy + y^2 - 2 = 0$$

and trace the conic. 12

Or

Rotate the axes of co-ordinate to get rid of the xy-term from the equation :

$$x^2 + 2\sqrt{3}\,xy + 3y^2 + 2\sqrt{3}\,x - 2y = 0$$

and trace the conic. 12

(*b*) Find an equation of a hyperbola with vertices (2, 4) and (10, 4) and foci 10 units apart. 5

(*c*) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. 5

(*d*) Let $\vec{A}$ and ϕ be the functions given by :

$$\vec{A} = 2yz\hat{i} - x^2y\hat{j} + xz^2\hat{k}$$

and $\phi = 2x^2yz^3$,

find $\vec{A} \cdot \nabla\phi$ and $\nabla \times \nabla\phi$. 5

(*e*) Let $\vec{r}$ be the radius vector given by :

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k};\ (x, y, z) \in R^3,\ (x, y, z) \neq (0, 0, 0).$$

Find ∇r^n, where $r = |\vec{r}|$. 5