This question paper contains 5 printed pages.

5129 *Your Roll No.* .....

B.Sc. (Prog.) / II **B**

OPERATIONAL RESEARCH

Paper OR-201 : Optimization

(Admissions of 2005 and onwards)

Time : 3 hours *Maximum Marks*: 112

(Write your Roll No. on the top immediately on receipt of this question paper)

Attempt any five questions.

1. (*a*) State the different types of models used in O.R. Explain briefly the general methods for solving these OR models. 10

(*b*) The Reddy Mikks Company owns a small paint factory that produces both interior and exterior house paints for wholesale distribution. Two basic raw materials, A and B, are used to manufacture the paints. The maximum availability of A is 6 tons a day; that of B is 8 tons a day. The daily requirements of the raw materials per ton of interior and exterior paints are summarized in the following table:

P. T. O.

	Tons of Raw Material per Ton of Paint		Maximum Availability (Tons)
	Exterior	Interior	
Raw Material A	1	2	6
Raw Material B	2	1	8

A market survey has established that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The survey also shows that the maximum demand for interior paint is limited to 2 tons daily.

The wholesale price per ton is Rs. 3000 for exterior and Rs. 2000 for interior paint.

How much interior and exterior paints should the company produce daily to maximize gross income? (Use graphical method to find the solution.) 12½

2. (*a*) Let $a_1, a_2, \ldots, a_r$ be a basis for E^n and $b \neq 0$ be any vector in E^n such that:

$$b = \sum_{i=1}^{r} \lambda_i a_i ; \quad \lambda_k \neq 0$$

then prove that $a_1, a_2, \ldots, a_{k-1}, b, a_{k+1}, \ldots, a_r$ is also a basis for E^n. 11

(*b*) Find all possible basic solutions for the following set of linear equations:

$$x_1 + 2x_2 + x_3 = 4$$
$$3x_1 + x_2 + 5x_3 = 5. \quad 11$$

3. (*a*) Define a convex set.

Prove that the set,

$$x = \left[(x_1, x_2) / 3x_1^2 + 5x_2^2 \leq 15\right]$$

is a convex set. 6½

(*b*) Define slack, surplus and artificial variables. 6

(*c*) Solve the following set of linear equations by the simplex method:

$$x_1 + x_2 = 5$$
$$2x_1 - x_2 = 1. \quad 10$$

4. (*a*) What is the significance of duality in linear programming? Describe the general rules for writing the dual of a linear programming problem. 7

(*b*) Write the dual of the following LPP:

Max $z = 4x_1 + 9x_2 - 7x_3$

subject to:

$$x_1 + x_2 + 4x_3 \leq 15$$
$$x_1 + x_2 + x_3 \geq 17$$
$$2x_1 + 3x_3 = 10$$
$$x_1, x_2 \geq 0, \ x_3 \text{ is unrestricted.} \quad 8½$$

P. T. O.

(*c*) Give a brief outline of the Dual Simplex Method. 7

5. (*a*) Solve the following Cost Minimizing Transportation Problem:

From \ To	D_1	D_2	D_3	D_4	Capacity
R_1	5	7	13	10	700
R_2	8	6	14	13	400
R_3	12	10	9	11	800
Demand	200	600	700	400	

11 1/2

(*b*) Solve the assignment problem whose cost matrix is given below:

	A	*B*	*C*	*D*
I	1	4	6	3
II	9	7	10	9
III	4	5	11	7
IV	8	7	8	5

11

6. What is integer linear programming? How does integer linear programming problem differ from the ordinary linear programming problem? What is the need for integer programming problem?.

Solve the following integer programming problem:

Max $Z = 3x_1 + x_2 + 3x_3$

Subject to:

$$-x_1 + 2x_2 + x_3 \leq 4$$
$$4x_2 - 3x_3 \leq 2$$
$$x_1 - 3x_2 + 2x_3 \leq 3$$

x_1, x_2 and x_3 all are non-negative integers. 22 1/2

7. (*a*) Solve the following LPP by the simplex method:

Max $Z = 3x_1 + 5x_2 + 4x_3$

Subject to:

$$2x_1 + 3x_2 \leq 8$$
$$2x_2 + 5x_3 \leq 10$$
$$3x_1 + 2x_2 + 4x_3 \leq 15$$

and $x_1, x_2, x_3 \geq 0$

Find the ranges in which c_2 and b_2 can be changed while maintaining optimality and feasibility of current optimal solution. (Symbols have their usual meanings.) 15 1/2

(*b*) What is 'degeneracy'? Explain any *one* method to resolve degeneracy in LPP. 7

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