This question paper contains 4+1 printed pages]

Your Roll No.....

5193E

B.Sc. (Physical Science)/IV Sem. B

Paper MAPT-404

MATHEMATICS—IV (Differential Equations)

(Admission of 2010 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *two* parts from each question.

All questions are compulsory.

Section A

1. (*a*) Solve the initial value problem : 6½

$$\frac{1 + 8xy^{2/3}}{x^{2/3}y^{1/3}}\,dx + \frac{2x^{4/3}y^{2/3} - x^{1/3}}{y^{4/3}}\,dy = 0,\ y(1) = 8.$$

(*b*) Solve : 6½

$$(4xy^2 + 6y)\,dx + (5x^2y + 8x)\,dy = 0.$$

(*c*) Solve : 6½

$$p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0,\ p = \frac{dy}{dx}.$$

P.T.O.

2. (*a*) Solve : 6½

$$(D^2 + 2)y = x^2\ e^{3x} + e^x \cos 2x, \quad \text{where } D \equiv \frac{d}{dx}.$$

(*b*) Solve : 6½

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2.$$

(*c*) Prove that two solutions f_1 and f_2 of the second order homogeneous linear differential equation :

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)\,y = 0,$$

where a_0, a_1, a_2 are continuous real functions on a real interval $a \le x \le b$ and $a_0(x) \neq 0$ for any $x \in [a, b]$, are linearly dependent on $[a, b]$ if and only if the Wronskian of f_1 and f_2 is zero, for all $x \in [a, b]$. 6½

3. (*a*) Find the general solution of :

$$(2x+1)(x+1)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = (2x+1)^2,$$

given that $y = x$ and $y = (x+1)^{-1}$ are linearly independent solutions of the corresponding homogeneous equation. 6½

(*b*) Given that $y = x$ is a solution of :

$$(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0,$$

Find a linearly independent solution by reducing the order. Write the general solution also. 6½

(*c*) A large tank initially contains 50 gal. of brine in which there is dissolved 10 lb of salt. Brine containing 2 lb of dissolved salt per gallon flows into the tank at the rate of 5 gal/min. The mixture is kept uniform by stirring and the stirred mixture simultaneously flows out at the slower rate of 3 gal/min. How much salt is in the tank at any time $t > 0$? 6½

4. (*a*) Solve : 6½

(*i*) $4\frac{dx}{dt} + 9\frac{dy}{dt} + 44x + 49y = t$

(*ii*) $3\frac{dx}{dt} + 7\frac{dy}{dt} + 34x + 38y = e^t$.

(*b*) Solve : 6½

$$\frac{dx}{x(y^2 - z^2)} = \frac{dy}{-y(z^2 + x^2)} = \frac{dz}{z(x^2 + y^2)}.$$

(*c*) Solve : 6½

$$(y^2 + yz)\,dx + (zx + z^2)\,dy + (y^2 - xy)\,dz = 0.$$

Section B

5. (*a*) Eliminate the arbitrary function f from the equation :

$$f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$$

to write the required partial differential equation. 5½

(*b*) Find the general integral of :

$$px(x + y) = qy(x + y) - (x - y)\,(2x + 2y + z).$$ 5½

(*c*) Find the complete integral of :

$$p^2y(1 + x^2) = qx^2.$$ 5½

6. (*a*) Find the complete integral of :

$$(p^2 + q^2)\,y = qz.$$ 6

(*b*) Show that the equations :

$$f(x, y, p, q) = 0, \quad g(x, y, p, q) = 0,$$

are compatible if :

$$\frac{\partial(f, g)}{\partial(x, p)} + \frac{\partial(f, g)}{\partial(y, q)} = 0$$

Also show that the equations :

$$p = P(x, y) \text{ and } q = Q(x, y)$$

are compatible if :

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$ 6

(*c*) Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form. 6