[This question paper contains 5 printed pages.]

1926 *Your Roll No.* ....

B.Sc. (Prog.) / II **E**

MA 201 – MATHEMATICS – I

(Calculus and Geometry)

(For Physical Sciences / Applied Sciences)

(Admissions of 2005 and onwards)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *sections are compulsory.*
There are a total of **five** *questions.*
Attempt any **two** *parts in each question of Section* **I** *and* **II.** *In Section* **III** *attempt* **four** *parts in question* **5,** *part (a) being compulsory.*

SECTION I

1. (a) (i) Find the equation of the circle described on the join of the points $(1 + i)$ and $(2 - i)$ as the extremities of one of its diameters. (7)

(ii) Add the complex numbers $z_1 = 4 + 3i$ and $z_2 = -2 + i$ and represent their sum geometrically. Also represent the difference of complex numbers $2-3i$ and $4+2i$ geometrically. (5½)

(b) (i) Solve the equation: $x^4 - 8x^3 + 21x^2 - 20x + 5 = 0$ given that the sum of two roots is equal to the sum of other two. (7)

(ii) If α, β, γ are the roots of $x^3 - qx - r = 0$ find the values of

(a) $\sum \frac{\beta+\gamma}{\alpha^2}$ (b) $\sum \alpha^4$ (5½)

(c) (i) Solve the equation $x^9 + x^5 + x^4 + 1 = 0$ (7)

(ii) If α, β are the roots of $x^2 - 2x + 4 = 0$ prove that $\alpha^n + \beta^n = 2^{n+1} \cos\frac{n\pi}{3}$ and hence evaluate $\alpha^6 + \beta^6$. (5½)

SECTION II

2. (a) (i) Using ε-δ approach show that

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = 4$$ (5)

(ii) Show that the function f defined as $f(x) = |x - 1| + |x + 1| \forall x \in R$ is not derivable at the points $x = -1$ and $x = 1$ and is derivable at every other point. (5)

(b) (i) State Darboux's theorem. Explain it with the help of an example. (5)

(ii) Prove that $f(x) = \frac{1}{x}$ is not uniformly continuous on $]0,1]$. (5)

(c) (i) Discuss the validity of the Rolle's theorem for $f(x) = (x - a)^m (x - b)^n$ in $[a,b]$; m,n being positive integers. (5)

(ii) Find the points of inflexion of the curve

$$y = \frac{x}{1+x^2}$$ (5)

3. (a) Find the asymptotes of the curve

$$y^3 - x^2y - 2xy^2 + 2x^3 - 3xy + 2x^2 + x - 2y + 1 = 0$$ (10)

(b) (i) Determine the position and the nature of double points on the curve :

$$y(y - 6) - x^2(x - 2)^2 + 9 = 0$$ (5)

(ii) Trace the curve $y = a\sin(3\theta)$. (5)

(c) Trace the following curve $y^2(a^2 - x^2) = x^4$. (10)

4. (a) Find the reduction formula for $\int x^n \sqrt{(a^2 - x^2)}\, dx$ (n being a positive integer). (10)

(b) Find the volume of solid obtained by revolving one all of the cardioide $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about x-axis. (10)

(c) Find the entire length of the cardioid $r = a(1 + \cos\theta)$. (10)

SECTION III

Attempt **four** *parts of this question, including* **(a)** *which is compulsory.*

5. (a) Rotate the coordinate axes to remove xy term from the equation.

$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$, and trace the conic. (12)

Or

Rotate the coordinate axes to remove xy term from the equation.

$52x^2 - 72xy + 73y^2 + 40x + 30y - 73 = 0$, and trace the conic. (12)

(b) Find the equation for the ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$. (5)

(c) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is time. Find the components of its velocity and acceleration at $t = 1$ in the direction $2\hat{i} + 2\hat{j} - \hat{k}$. (5)

(d) If $\bar{A} = yz^2\hat{i} - 3xz^2\hat{j} + 2xyz\hat{k}$, $\bar{B} = 3x\hat{i} + 4z\hat{j} - xy\hat{k}$ and $\varphi = xyz$, find $(\bar{A} \times \bar{\nabla})\varphi$ and $\bar{A} \times \bar{\nabla}\varphi$. (5)

(e) If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\bar{r}|$, Prove that $\bar{\nabla} \cdot \left(\frac{\bar{r}}{r}\right) = \frac{2}{r}$. (5)