[This question paper contains 5 printed pages.]

1927 *Your Roll No.* ....

B.Sc. Prog. / II **E**

MA-202 – MATHEMATICS – II – Algebra & Differential Equations

(For Physical Sciences / Applied Physical Sciences)

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each question.*
All *questions are compulsory.*

UNIT – I

1. (a) Define a group. Show that

$$\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in Z \text{ and } ad - bc = 1\right\}$$ is a group under matrix multiplication. (7½)

(b) Let G be a group, $a \in G$ and C(a) be the centralizer of 'a'. Prove that $C(a) = C(a^{-1})$. (7½)

P.T.O.

(c) Write elements of the group U(8). Show that all of its proper subgroups are cyclic. Is U(8), also, a cyclic group ? (7½)

2. (a) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 3 & 5 & 4 \end{pmatrix}$ and

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 3 & 2 & 6 & 4 & 5 \end{pmatrix}$$

(i) Compute $\alpha\beta$ and $\beta\alpha$

(ii) Find $o(\alpha\beta)$ and $o(\beta\alpha)$ (7½)

(b) Let H be a subgroup of a group G and let $a, b \in G$. Show that

$$aH = bH \Leftrightarrow a^{-1}b \in H$$ (7½)

(c) Let $G = \langle a \rangle$ s.t. $o(a) = 15$. Let $H = \langle a^5 \rangle$ be a subgroup of G. Find all distinct left cosets of H in G. (7½)

3. (a) Prove that a factor group of an Abelian group is Abelian. (7½)

(b) Show that the alternating subgroup A_3 of the group S_3 is a normal subgroup of S_3. (7½)

(c) Let G be a group and let $a \in G$. Prove that

$\langle a^{-1} \rangle = \langle a \rangle$. (7½)

UNIT – II

4. (a) Solve :

(i) $\left(y^4 + 2y\right)dx + \left(xy^3 + 2y^4 - 4x\right)dy = 0$

(ii) $xp^2 - yp - y = 0,\ p = \frac{dy}{dx}$ (6,5½)

(b) Solve by the method of variation of parameters :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x$$ (11½)

(c) Given that $y = x + 1$ is a solution of

$$\left(x+1\right)^2 \frac{d^2y}{dx^2} - 3\left(x+1\right)\frac{dy}{dx} + 3y = 0,$$

find a linearly independent solution by reducing the order. Also write its general solution.

(11½)

5. (a) Solve :

$$\frac{dx}{dt} + 2\frac{dy}{dt} - 2x + 2y = 3e^t$$

$$3\frac{dx}{dt}+\frac{dy}{dt}+2x+y=4e^{2t}$$ (11)

(b) Solve :

$$\left(2x^2+2xy+2xz^2+1\right)dx+dy+2zdz=0$$ (11)

(c) A 16 *lb* weight is attached to the lower end of a coil spring suspended from the ceiling, the spring constant of the spring being 10 *lb/ft*. The weight comes to rest in its equilibrium position. Beginning at $t = 0$ an external force given by $F(t) = 5 \cos 2t$ is applied to the system. Determine the resulting motion if the damping force in pounds is numerically equal to $2\frac{dx}{dt}$, where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. (11)

UNIT – III

6. (a) Find the general integral of

$$(y + zx)p - (x + yz)q = x^2 - y^2,$$

where $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$. (11)

(b) Find the complete integral of

$$2(y + zq) = q(xp + yq),$$

where $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$. (11)

(c) Show that the equations $f(x, y, p, q) = 0$ and $g(x, y, p, q) = 0$ are compatible if

$$\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0 .$$

Also show that the equations $p = P(x,y)$, $q = Q(x,y)$ are compatible if

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x},$$

where $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$. (11)