This question paper contains 3 printed pages. *Your Roll No. .....*

Sl. No. of Ques. Paper : **1845** **GC-3**

Unique Paper Code : **42364303**

Name of Paper : **Optimization Techniques**

Name of Course : **B.Sc. (Prog.) Mathematical Sciences (CBCS)**

Semester : **III**

Duration : : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions. All questions carry equal marks.
Simple calculator is allowed.

1.(a) By using the definition of a convex function, show that the function $f(X) = 2x_1^2 + x_2^2$ is convex over R^2. (7)

(b) Determine whether the following functions are positive definite/semi-definite or negative definite/semi-definite:

$$f(X) = -x_1^2 - x_2^2 - 4x_3^2 + x_1x_2 - 2x_2x_3 \quad (5)$$

(c) Explain the differences between linear programming and dynamic programming approach of solving an optimization problem. (3)

2. (a) Suppose a manufacturing firm encounters price elasticity in selling three products. Let the unit cost of producing the products 1,2 & 3 be Rs. 25/-, Rs. 10/- & Rs. 15/- respectively and the prices required in order to be able to sell x_1, x_2 *and* x_3 are $\left(35 + 100\, x_1^{-\frac{1}{3}}\right)$, $\left(15 + 40x_2^{-\frac{1}{4}}\right)$, *and* $(20 + 50x_3^{-1/2})$. Formulate and solve it as a nonlinear programming problem (NLPP) for determining the number of units of each product to be produced by the firm to maximize the profit.

(6)

(b) Solve the following NLPP by using method of Lagrangian multipliers:

$$\text{Minimize } Z = x_1^2 + x_2^2 + x_3^2$$
$$\text{subject to } x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$
$$x_1, x_2, x_3 \geq 0$$

(9)

P. T. O.

3. (a) Write down the quadratic form whose associated matrix is

$$\begin{bmatrix} 2 & -3 & 1 \\ -3 & 4 & 2 \\ 1 & 2 & -6 \end{bmatrix}$$ (3)

(b) A firm produces two products A and B. The production of these products uses 1 hour each of a certain machine time per unit. The total machine hour at the firm's disposal is 400 hours. As per the Market survey the firm can sell a maximum of 240 units of A and 300 units of B. The net profit from the sale A and B are Rs.800/- and Rs. 400/- respectively. The manager's priorities are described as follows:

- He wants to avoid any underutilisation of the normal production capacity.
- He wants to sell maximum units of A and B in the market. Since the net profit from A is twice from net profit from B the manager has twice desire to sell product A over B.
- He wants to minimize the overtime operation of the plant as much as possible.

Formulate the above problem as a goal programming problem and then solve it using graphical method. (12)

4. Derive the Karush Kuhn Tucker (KKT) conditions for solving a general NLPP involving m inequality constraints and n variables. State the necessary and sufficient conditions for having an optimum solution and use those conditions to solve the following problem:

$$\begin{aligned} &Max\, Z = 2x_1 + x_2 - x_1^2 \\ &subject\ to \quad 2x_1 + 3x_2 \le 6 \\ &\qquad\qquad 2x_1 + x_2 \le 4 \\ &\qquad\qquad x_1, x_2 \ge 0 \end{aligned}$$

(15)

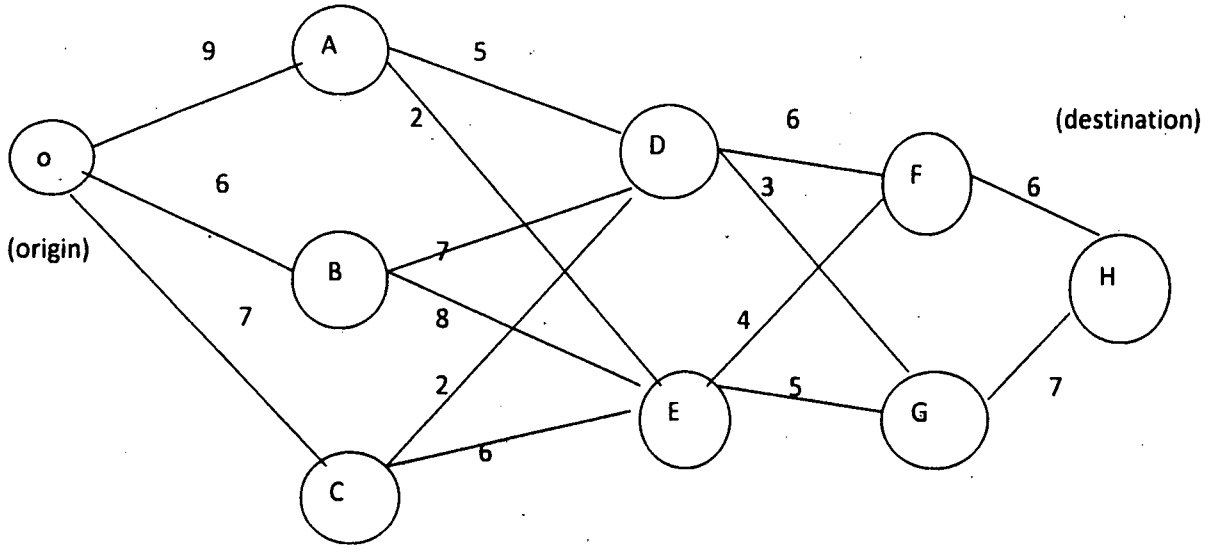
5. (a) Describe the importance of goal programming and differentiate between preemptive and non-preemptive goal programming problem. What is the role of deviational variables in a goal programming problem. (6)

(b) Solve the following quadratic programming problem (QPP) by Beale's Method:

$$\begin{aligned} &Max\, Z = 2x_1 + 3x_2 - x_1^2 \\ &subject\ to \quad x_1 + 2x_2 \le 4 \\ &\qquad\qquad x_1, x_2 \ge 0 \end{aligned}$$

(9)

6. Consider the following network where numbers on the links represent the actual distance between the pair of nodes connected by that link. Find the shortest route from origin to destination by using dynamic programming approach.



Write down the stages, states, recursive equation and dynamic programming formulation of the problem clearly. (15)

7 (a) What is a Hessian matrix and how it is used for testing whether a function is convex or concave.

(5)

(b) Find the maximum of the following function

$$f(x) = -(x-3)^2 \quad ; \; 2 \leq x \leq 4$$

Using (i) one dimensional search procedure €=0.01 and (ii) Dichotomous method. Δ=0.1

(Show maximum three iterations only) (10)