

This question paper contains 4+2 printed pages]

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S. No. of Question Paper : 63

Unique Paper Code : 235161

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Name of the Paper : Mathematics and Statistics (MACT-303)

Name of the Course : B.Sc. (Hons.) Botany/Zoology/Biological Sciences/B.Sc. Life Sciences

Semester : III/I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are *three* sections in this question paper.

Attempt any *two* questions from each section.

Students are allowed to use simple calculator.

Section I

1. (a) For what value of k is the function :

5½

$$f(x) = \begin{cases} x^2 + 5, & x \leq 2 \\ k - 4x, & x > 2 \end{cases}$$

continuous at $x = 2$?

- (b) Out of 300 people in an office, 140 drink coffee and 180 drink tea. If 45 drink neither, find out how many drink both. Also illustrate this fact by a Venn diagram. 5½

- (c) Let x be a multiple of 3 (i.e. 0, 3, 6, 9, 12) and $f(x)$ be the remainder obtained on dividing x by 5. Is $f(x)$ a periodic function of x ? If yes, what is the period ? Plot the graph of the function. 5½

P.T.O.

2. (a) Is the sequence

$$\left\langle \frac{\sin^2 n}{n} \right\rangle$$

convergent ? If yes, what is its limit ?

5½

- (b) Find the sum of the infinite geometric series :

5½

$$5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$$

- (c) (i) Differentiate $\log_e(\log_e(x))$ w.r.t. x .

(ii) If

$$y = 2 - \sin x + \sec x,$$

$$\text{find } \frac{dy}{dx} \text{ at } x = \frac{\pi}{6}.$$

5½

3. (a) Expand $e^{-x} + e^{2x}$ in ascending powers of x .

5½

- (b) Write the first five terms of the sequence given by the recursion formula :

5½

$$a_1 = -2, a_2 = 1, a_{n+2} = \frac{a_{n+1}}{a_n}.$$

- (c) Evaluate :

5½

$$\int_2^{10} \frac{1}{(2x+7)\sqrt{2x+7}} dx.$$

4. (a) Evaluate any two of the following :

6

$$(i) \int_e^{e^3} \frac{1}{x \log x} dx$$

(ii) $\int \cos \sqrt{x} \, dx$

(iii) $\int e^x (x^2 + 2x) dx$.

(b) If

$$A = \{x : x \geq 8\} \text{ and } B = \{x : x < 24\},$$

then find

$$A \cup B^C, A \cap B \text{ and } (A \cup B) \setminus (A \cap B)$$

where B^C denotes the complement of B .

5

(c) Assume that a given population grows according to the rule

$$P(t) = 50000 + 50 t^2,$$

where the time t is measured in hours. Find the average growth rate during the time interval $t = 2$ to $t = 8$.

5½

Section II

5. (a) If

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} p & 1 \\ q & -1 \end{bmatrix} \text{ and } (A + B)^2 = A^2 + B^2;$$

determine the values of p and q .

6

(b) If

$$g(x) = x^2 + 5x - 7 \text{ and } A = \begin{bmatrix} 3 & 1 \\ -1 & -2 \end{bmatrix},$$

find $g(A)$.

5

P.T.O.

(4)

6. (a) If

$$A = \begin{bmatrix} 14 & 1 & -1 \\ 3 & 2 & 15 \\ 0 & 8 & 14 \end{bmatrix} \text{ and } B = \begin{bmatrix} -4 & 21 & 6 \\ 13 & 4 & -1 \\ 2 & 0 & 2 \end{bmatrix},$$

find matrix X such that

$$A + B - 2X = 0.$$

(b) If

$$A = \begin{bmatrix} 1 & 2 & 5 \\ -4 & 1 & 6 \\ 8 & 4 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix},$$

compute AB and BA. Is it true that $AB = BA$?

7. (a) Find the image of the point $(-2, 4)$ under rotation through an angle 45° in the counterclockwise direction.

(b) If

$$A = \begin{bmatrix} 0 & 3 \\ -2 & 5 \end{bmatrix},$$

find k so that

$$kA^2 = 5A - 6I_2.$$

4.

(c) Express the following as a set of linear equations :

$$(i) \begin{bmatrix} 1 & -y & 2 \\ 3 & 15 & 3x \\ z & 3 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ 2 \end{bmatrix}$$

(5)

63

$$(ii) \begin{bmatrix} 1 & y & -9 \\ 3 & 0 & x \end{bmatrix} \begin{bmatrix} z & 0 \\ 1 & -1 \\ 2 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 6 & 8 \end{bmatrix}$$

Section III

8. (a) In a botanical lab there are 16 plants. A carcinogen is sprayed uniformly. Find the probability of getting half of the total plants mutated, assuming that the chances of mutation in each plant is 50%.

s

75

- (b) Find the mean and standard deviation of the two observations $a + b$ and $a - b$.

5

9. (a) The mean of a certain number of observations is 40. If two more items with values 50 and 64 are added to this data, the mean rises to 42. Find the number of items in the original data.

5½

- (b) The marks of 5 students are given in Mathematics and Physics. Find the coefficient of correlation between them :

5

5

Marks in Mathematics	2	8	5	4	6
Marks in Physics	1	7	6	2	4

: neither,

5½

: obtained

iod ? Plot

5½

P.T.O.

10. (a) Fit a straight line to the following data with x as independent variable and y as dependent variables :

5

x	1	2	3	4	5	6
y	2.4	3	3.6	4	5	6

P.T.O.

- (b) In an intelligence test administered to 1000 children, the average score is 42 and S.D. is 24. Assuming normal distribution, find the number of children whose scores lie between 20 and 40. 5
11. (a) The means of two samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the population of standard deviation 2.5 inches ? 5
- (b) In a certain factory manufacturing razor blades, there is a small chance $1/500$ for any blade to be defective. The blades are in the packets of 10. Use Poisson distribution to find the probability that the packet contains : 5
- (i) No defective blade
- (ii) At least one defective blade.